

Math 464 - Fall 10 - Project II

The goal of this project is to illustrate the weak and strong law of large numbers and the central limit theorem. Choose a random variable X which has finite variance but does not have a normal distribution. Let X_1, X_2, X_3, \dots be independent random variables, all of which have the same distribution as X . Let

$$\bar{X}_n = \frac{1}{n} \sum_{j=1}^n X_j$$

- (a) For $n = 10, 100$ and 1000 , generate a large number of samples of \bar{X}_n and use them to plot histograms of $\bar{X}_{10}, \bar{X}_{100}$, and \bar{X}_{1000} . Note that to generate each sample of \bar{X}_n you have to generate n samples of X .
- (b) Which law of large numbers (weak or strong) is illustrated by your three histograms in (a)? Explain your answer.
- (c) Now generate a single sequence $X_1, X_2, \dots, X_{10000}$. For each $n = 1, 2, \dots, 10000$, compute \bar{X}_n for this sequence and then plot \bar{X}_n as a function of n .
- (d) Which law of large numbers (weak or strong) is illustrated by your graph in (c)? Explain your answer.
- (e) Now let

$$Y_n = \frac{1}{\sqrt{n}} \sum_{j=1}^n (X_j - \mu)$$

where μ is the mean of X . For $n = 10, 100$ and 1000 , generate a large number of samples of Y_n and use them to plot histograms of Y_{10}, Y_{100} , and Y_{1000} .

- (f) The central limit theorem gives an approximation to the distributions of Y_{10}, Y_{100} , and Y_{1000} . What is it? Explain how your histograms in (e) illustrate the central limit theorem.