Sample Exam 1 - Math 464 - Fall 10 -Kennedy

The questions on this sample exam are meant to be representative of the questions that will be on the exam. However, if a topic does not appear on the sample exam that does not mean it will not appear on the exam. This sample exam is probably slightly too long for a real exam.

1. Let A and B be events with

$$\mathbf{P}(A) = 0.1, \qquad \mathbf{P}(B|A) = 0.7, \qquad \mathbf{P}(B|A^c) = 0.2$$

(a) Find $\mathbf{P}(B)$.

(b) Find $\mathbf{P}(A|B)$.

(c) Are A and B independent ? Justify your answer. A simple yes or no gets no credit.

2. Die A has twelve faces, 9 of which are green and 3 of which are white. Die B has twelve faces, 6 of which are green and 6 of which are white. I flip a fair coin. If it is heads, I roll die A, and for tails I roll die B.

(a) Find the probability green turns up, i.e., the face on the top of the die is green.

(b) If green turns up, what is the probability die A was rolled?

3. Let X be a discrete RV with a Poisson distribution whose mean is 2.

(a) Find $\mathbf{P}(X \leq 1|X \leq 2)$.

(b) Let $Y = 5 - 2X$. Find the mean and variance of Y.

4. I flip a fair coin n times and let X be the number of heads minus the number of tails.

(a) Find the p.m.f. of X. (In the first version of the sample exam I mistakenly asked for the "density" of X.) Hint: X is not binomial, but it is closely related to a random variable whose distribution is binomial.

(b) Find the probability X is strictly greater than zero.

5. I have 10 one-dollar bills, 6 five-dollar bills and 2 ten-dollar bills.

(a) If I randomly arrange the bills in a row, what is the probability that as I look at them from left to right I first see all the one-dollar bills, then all the five-dollar bills and finally the ten-dollar bills?

(b) If I put all the bills in a hat and draw three, what is the probability the total value is no greater than 7$?

(c) I am going to give all of the one-dollar bills to three friends. In how many ways can I do this if the bills are identical, and I don't care how many bills each friend gets?

(d) In how many ways can I give all 18 of the original bills to my friends? (Assume that bills with the same value are identical.)

6. We roll a six-sided die until we get a 6. Let X be the sum of all the numbers that we came up. For example, if the rolls are $2, 4, 3, 6$, then $X = 2+4+3+6$. Find the mean X. (In the first version of the sample exam I asked for the variance as well. This is too complicated.)

7. Consider a simple trial with two outcomes (success and failure) which is repeated $2n$ times with the repetitions being independent of one another. Let X be the total number of successes out the $2n$ reps, and Y the number of successes out of the first n reps.
(a) Find the joint density of X and Y, i.e., find a formula for $\mathbf{P}(X = k, Y = l)$. Since they are not independent this is not so easy. There are at least two ways to do this. Here are hints for each of them.

Combinatorial approach: Think of the sample space as sequences of S's and F's. What does $X = k, Y = l$ tell you about the sequence?

Using independence: Let Z be the number of successes in the last n trials. Write X in terms of Y and Z.

(b) Compute $\mathbf{E}[XY]$.

(c) Are X and Y independent? Justify your answer.

2