Sample Exam 2 - Math 464 - Fall 10 -Kennedy

The questions on this sample exam are meant to be representative of the questions that will be on the exam. However, if a topic does not appear on the sample exam that does not mean it will not appear on the exam. This sample exam is probably slightly too long for a real exam.

1. Let X have a gamma distribution with $\lambda = 2$, $w = 3$. Let $Y = 3X$. Show that Y has a gamma distribution and find the values of λ and w for Y. (Hint: how is the moment generating function of Y related to that of X?)

2. Let Z have a standard normal distrbution. (So its mean is 0 and its variance is 1.) Let $X = e^{-Z}$.
(a) Find the mean and variance of X.
(b) Find the probability density function (pdf) for X.

3. Let X and Y be continuous random variables with joint pdf

$$f_{X,Y}(x, y) = \frac{3}{2}(x^2 + y^2), \quad 0 \le x \le 1, 0 \le y \le 1$$

Outside of $0 \le x \le 1, 0 \le y \le 1$, $f_{X,Y}(x, y) = 0$.
(a) Find the marginal densities of X and Y
(b) Are X and Y independent?

4. X and Y are independent continuous random variables. X is uniformly distributed on $[-1, 1]$. Y has an exponential distribution with $\lambda = 1$.
(a) Let $Z = 2X + Y$. Find the mean and variance of Z.
(b) Compute the probability density function, $f_Z(z)$, of Z for $z \ge 2$.

5. (a) Let X be a standard normal. The odd moments of X are zero. Use the mgf to compute $\mathbf{E}[X^2], \mathbf{E}[X^4]$ and $\mathbf{E}[X^6]$. Hint: Before you go crazy computing derivatives, think about the power series expansion

$$e^{t^2/2} = 1 + \frac{t^2}{2} + \frac{1}{2}\left[\frac{t^2}{2}\right]^2 + \frac{1}{3!}\left[\frac{t^2}{2}\right]^3 + \cdots$$

(b) Let X, Y be independent random variables, each with the standard normal distribution. Let $Z = X^2 + Y^2$. Find the mean and variance of Z.

6. Random variables X and Y have joint cumulative distribution function (cdf)

$$F_{X,Y}(x,y) = \begin{cases} [\frac{1}{\pi}\tan^{-1}(x) + c](1 - e^{-y}), & \text{if } y \geq 0 \\ 0, & \text{if } y < 0 \end{cases}$$

where c is some constant.
(a) Find the value of c.
(b) Find the joint probability density function (pdf) for X, Y. (Note that you don't need to know the value of c to do this.)

7. The joint pdf of X and Y is

$$f_{X,Y}(x,y) = \frac{1}{2\pi}\exp(-\frac{1}{2}x^2 - xy - y^2), \quad -\infty < x < \infty, -\infty < y < \infty$$

Define new random variables by

$$\begin{aligned} U &= X + Y \\ V &= 2Y \end{aligned}$$

(a) Are X and Y independent?
(b) Find the joint density of U, V.
(b) Are U and V independent?