Review questions for Exam 1 - Math 464 - Fall 18 -Kennedy

These questions are meant to be representative of the questions that will be on the exam. However, if a topic does not appear here that does not mean it will not appear on the exam. These nine problems would be too long for a reasonable exam.

1. Let X be a discrete RV with a Poisson distribution whose mean is 3.

(a) Find $P(X \geq 1 | X \leq 2)$.

(b) Let $Z = 2X + 5$. Find the mean and variance of Z.

2. A and B are events with $\mathbf{P}(A) = 0.1, \quad \mathbf{P}(B|A) = 0.7, \quad \mathbf{P}(B|A^c) = 0.2$.

(a) Find $\mathbf{P}(B)$.

(b) Find $\mathbf{P}(A|B)$.

(c) Are A and B independent ? Justify your answer.

3. Each day a weatherman makes one of three predictions: "rain", "no rain", or "possibility of rain." The percentages of the time he makes each prediction are 10%,75%,and 15% respectively. If the weather forecast is for "rain," the probability it will rain is 70%. If the forecast is "no rain," the probability it will rain is 20%. If the forecast is for "possibility of rain", the probability of rain is 50%.
(a) Find the percentage of days on which it rains.

(b) Suppose it did not rain yesterday. What is the probability the forecast for yesterday was for "no rain?"

4. Let X and Y be independent random variables. They are identically distributed and both have the geometric distribution with $p = 1/3$.

(a) What is the joint pmf of X and Y ?

(b) Let $W = X + Y$. Find $\mathbf{P}(W = 3)$.

(c) Let $Z = X - Y$. Find $\mathbf{P}(Z = 0)$.

(d) Let $Z = X - Y$. Find the mean and variance of Z.

(e) Find $\mathbf{E}[XY^2]$.

5. N is a discrete random variable with range $\{1, 2, 3, 4\}$. It has the uniform distribution. An unfair coin has probability p of heads. We flip the coin N times and let X be the number of heads we get.

(a) Find $\mathbf{P}(X = 3)$.

(b) Find the mean and variance of X.

6. N is a discrete random variable with range $\{1, 2, 3, 4\}$. It has the uniform distribution. An unfair coin has probability p of heads. We flip the coin N times and let X be the number of heads we get.

(a) Find $\mathbf{P}(X = 3)$.

(b) Find the mean and variance of X.

7. Die A has twelve faces, 9 of which are green and 3 of which are white. Die B has twelve faces, 6 of which are green and 6 of which are white. I flip a fair coin. If it is heads, I roll die A, and for tails I roll die B.

(a) Find the probability the face on the top of the die is green.

(b) If green turns up, what is the probability die A was rolled?

8. I have 10 one-dollar bills, 6 five-dollar bills and 2 ten-dollar bills.

(a) If I randomly arrange the bills in a row, what is the probability that as I look at them from left to right I first see all the one-dollar bills, then all the five-dollar bills and finally the ten-dollar bills?

(b) If I put all the bills in a hat and draw three, what is the probability the total value is no greater than 7$?

(c) In how many ways can I give all of the one-dollar bills to 3 friends if the bills are considered to be identical?

9. Consider a simple trial with two outcomes (success and failure) which is repeated $2n$ times with the repetitions being independent of one another. Let X be the total number of successes out the $2n$ reps, and Y the number of successes out of the first n reps.

(a) Find the joint p.m.f.of X, Y. Here are hints for two different approaches:

Combinatorial approach: Think of the sample space as sequences of S's and F's. What does $X = k, Y = l$ tell you about the sequence?

Using independence: Let Z be the number of successes in the last n trials. Write X in terms of Y and Z.

(b) Compute $\mathbf{E}[XY]$.

(c) Are X and Y independent? Justify your answer.