Practice problems - Exam 2 - Math 464 - Fall 18

The questions on this sample exam are meant to be representative of the questions that will be on the exam. However, if a topic does not appear on the sample exam that does not mean it will not appear on the exam.

1. Let X and Y be independent random variables. They both have a gamma distribution with mean 3 and variance 3.

(a) Find the joint probability density function (pdf) of X, Y.

(b) Express $P(3X + Y \leq 3)$ as a double integral. Do not try to do the integral.

2. Let X have an exponential distribution with $E[X] = 1$. Let $Y = X^2 - 2$.
(a) Find the mean and variance of Y.
(b) Find the probability density function (pdf) for Y.

3. Let X and Y be continuous random variables with joint pdf

$$f_{X,Y}(x, y) = \frac{3}{2}(x^2 + y^2), \quad 0 \leq x \leq 1, 0 \leq y \leq 1$$

Outside of $0 \leq x \leq 1, 0 \leq y \leq 1$, $f_{X,Y}(x, y) = 0$.

(a) Find the marginal densities of X and Y

(b) Are X and Y independent?

4. Let X, Y be jointly continuous random variables with joint probability density function (pdf)

$$f_{X,Y}(x, y) = \begin{cases} 4xy, & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Let $Z = X + Y$. Compute $f_Z(z)$, the probability density function (pdf) for Z.

5. Let X and Y be independent random variables, each of which has a standard normal pdf. Let $Z = Y - X + 4$.

(a) Find the mean and variance of Z.

(b) Find the probability density function (pdf) of Z. Hint: this can be done with very little computation.

6. Random variables X and Y have joint cumulative distribution function (cdf)

$$F_{X,Y}(x,y) = \begin{cases} [\frac{1}{\pi}\tan^{-1}(x) + c](1 - e^{-y}), & \text{if } y \geq 0 \\ 0, & \text{if } y < 0 \end{cases}$$

where c is some constant.
(a) Are X and Y independent?
(b) Find the value of c.
(c) Find the joint probability density function (pdf) for X, Y. (Note that you don't need to know the value of c to do this.)

7. A RV X has a Laplace distribution if its pdf is

$$f_X(x) = \frac{1}{2}\lambda e^{-\lambda|x|}, \quad -\infty < x < \infty$$

where $\lambda > 0$ is a parameter.

(a) Compute the moment generating function of X.

(b) Find the mean and variance of X.

8. Let X_j be a sequence of independent, identically distributed random variables. Their common pdf is

$$f(x) = 4xe^{-2x}, x \geq 0$$

(It is zero for $x < 0$.) Let

$$\overline{X}_n = \frac{1}{n}\sum_{j=1}^{n} X_j$$

(a) Find the mean and variance of $\overline{X}_n$. Hint: the pdf $f(x)$ is in our catalog.
(b) For $n = 1000$, the probability that $\overline{X}_{1000}$ is in $[1, 1.1]$ is approximately given by

$$\int_a^b \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$$

Find a and b.

9. A random number generator produces random real numbers that are uniformly distributed between -1 and 1. I call it n times and let $\overline{X_n}$ be the average of the n numbers I get. As n goes to ∞, $P(|\overline{X_n}| \geq 0.01)$ goes to zero. Find n so that this probability is approximately 0.05.

For a standard normal RV Z, we have $P(Z \leq -2.576) = 0.005$, $P(Z \leq -2.326) = 0.01$, $P(Z \leq -1.960) = 0.025$, $P(Z \leq -1.649) = 0.05$.