Math 523a - Homework 6

1. Folland, problem 4, p 88.

2. Folland, problem 12, p 92.

3. Folland, problem 16, p 92.

4. Folland, problem 17, p 93.

5. Folland, problem 21, p 94.

6. Consider the vector space V of finite signed measures on some fixed measurable space $(X, \mathcal{M})$. In class we defined a norm on it by $||\mu|| = |\mu|(X)$.

   (a) Prove that V is complete in this norm.

   (b) Prove that

   $$||\mu|| = \sup \int f \, d\mu$$

   where the sup is over all measurable real valued f with $|f| \leq 1$ a.e.

7. Let $F(x)$ be a right continuous increasing function on $\mathbb{R}$ and let μ be the associated Borel measure. (So $\mu((a,b]) = F(b) - F(a)$.) Suppose that F has only a finite number of discontinuities (jumps) and in between the jumps it is C^1. (C^1 means that on the open intervals between the jumps it is differentiable and the derivative is a continuous function.) Let m be Lebesgue measure on the real line. The Lebesgue-Radon-Nikodym theorem says that there are measures λ and ρ such that $\mu = \lambda + \rho$ with $\rho \perp m$ and $\lambda << m$. And there is a non-negative measurable function f such that $\lambda = fm$. Give an explicit description of f and ρ. You may assume the fundamental theorem of calculus: if F is C^1 on (a,b) and $a < c < d < b$, then $\int_c^d F'(x) \, dx = F(d) - F(c)$.