Math 523a - Midterm - In class part

1. Let $(X, \mathcal{M}, \mu)$ be a measure space. Let $f \in L^1(X, \mathcal{M}, \mu)$

   (a) Show that if there is a constant $\epsilon > 0$ such that $f \geq \epsilon$ a.e., then $\mu(X) < \infty$.

   (b) Show that if $f > 0$ a.e., then μ is σ-finite. (Recall that σ-finite means that there is a countable collection of measurable sets X_n such that $X = \cup_{n=1}^{\infty} X_n$ and $\mu(X_n) < \infty$.)

2. Let $(X, \mathcal{M}, \mu)$ be a measure space and $f \in L^1(X, \mathcal{M}, \mu)$. Find

$$\lim_{n\to\infty} \int f(x) \exp(-n|f(x) - 1|) \, d\mu$$

   You should prove your result.

3. Let μ be a finite Borel measure on $\mathbb{R}^2$. Define a function on $\mathbb{R}$ by

$$G(t) = \mu(\{(x, y) : x + y > t\})$$

   Prove $G(t)$ is right continuous.