Math 523b - Homework 2

1. Problem 56, p. 135 in Folland.
2. Problem 60, p. 138 in Folland.
3. Problem 63, p. 138 in Folland.
4. Problem 67, p. 142 in Folland.
5. Problem 68, p. 142 in Folland.
6. Let X be a locally compact Hausdorff space. Let μ be a Borel measure on X. We say it is a Radon measure if it satisfies these three properties:

   (a) (local finiteness) For all compact sets K, $\mu(K) < \infty$.

   (b) (outer regularity) For all Borel sets E

   $$\mu(E) = \inf\{\mu(U) : U \textit{ open}, E \subset U\}$$

   (c) (inner regularity) For all Borel sets E

   $$\mu(E) = \sup\{\mu(K) : K \textit{ compact}, K \subset E\}$$

   Prove that if μ is such a measure, then $C_c(X)$, the continuous functions with compact support, are dense in $L^1(X, \mu)$.

7. Let X be a separable compact metric space. Prove that $C(X)$ (real or complex valued) is separable. Hint: Let x_k be a dense sequence in X and consider the algebra of functions generated by the functions

   $$f_{k,n} = \max\{0, 1 - n\rho(x, x_k)\}$$

   where ρ is the metric.