Math 523b - Homework 3

1. Problem 2, p. 154 in Folland.
2. Problem 9, p. 155 in Folland.
3. Problem 21, p. 160 in Folland.
4. Problem 22, p. 160 in Folland.
5. Problem 25, p. 160 in Folland.
6. Problem 34, p. 164 in Folland.
7. Problem 37, p. 165 in Folland.
8. Problem 39, p. 165 in Folland.
9. This problem is a special case of a general theorem we will prove later. Let c_0 denote the Banach space of sequences which converge to 0 with the norm
$$||(x_n)_{n=1}^{\infty}||_{\infty} = \sup_n |x_n|$$
Let l^1 be the Banach space of absolutely summable sequences with the norm
$$||(x_n)_{n=1}^{\infty}||_1 = \sum_{n=1}^{\infty} |x_n|$$
Our goal is to show that the dual of c_0 can be identified with l^1 in a natural way.
(a) Given $x = (x_n)_{n=1}^{\infty} \in l^1$, define a linear functional ϕ_x on c_0 by
$$\phi_x((y_n)_{n=1}^{\infty}) = \sum_{n=1}^{\infty} x_n y_n$$
Prove that ϕ_x is a bounded linear functional whose norm equals $||x||_1$. Thus the map $x \to \phi_x$ is an isometry of l^1 into the dual of c_0.
(b) Prove that this isometry is onto.