Math 523b - Homework 5

1. Folland problem 19, p. 192
2. Folland problem 20a, p. 192. Note that only part (a) is assigned.
3. Folland problem 32, p. 197
4. Folland problem 36, p. 199
5. Folland problem 39, p. 199
6. Folland problem 2, p. 215
7. Folland problem 4, p. 215
8. This problem gives an example to show that the measure associated with a postive linear functional on $C_c(X)$ by the Riesz representation theorem need not be regular. Let X be the plane with the following topology. A set is open if and only if its intersection with every vertical line is an open subset of that line. Show that X is a locally compact Hausdorff space. Let $f \in C_c(X)$. Show that there are only finitely many x such that $f(x, y) \neq 0$ for at least one y. Let $x_1, x_2, \cdots x_n$ be these x's. Define

$$F(f) = \sum_{j=1}^{n} \int_{-\infty}^{\infty} f(x_j, y)dy$$

Obviously F is a positive linear functional, and so is represented by a measure μ. Let E be the x-axis. Show that $\mu(E) = \infty$ but $\mu(K) = 0$ for every compact $K \subset E$.