Math 525a - Fall 17 - Homework 3

1. Problems 4(c) and 7(c) in section 2.1 of the text.

2. Problem 9 in section 2.1 of the text.

3. Let f be a real-valued function defined on $\mathbb{R}$. Suppose that for all x_0, $\lim_{x \to x_0}$ exists. Prove that for every interval $[a, b]$, f is bounded on the interval, i.e., there an M such that $|f(x)| \leq M$ for $x \in [a, b]$. Hint: use the previous problem to construct an open cover of $[a, b]$.

4. Problem 12 in section 2.1 of the text.

5. Let K be a compact subset of the real numbers, and let F be a closed subset of the real numbers. Suppose that K and F are disjoint. Prove there is an $\epsilon > 0$ such that for all $x \in K$ and all $y \in F$, $|x - y| \geq \epsilon$. Hint: Since F is closed, if $x \notin F$ there is an $\epsilon > 0$ such that $(x - \epsilon, x + \epsilon)$ is disjoint from F. Use this to construct an open cover of K.