Math 525a - Fall 17 - Homework 4

1. Let $f(x)$ and $g(x)$ be defined and bounded on a neighborhood of x_0. Prove

$$\limsup_{x\to x_0}[f(x)+g(x)] \le \limsup_{x\to x_0} f(x) + \limsup_{x\to x_0} g(x)$$

Give an example to show we can have $<$. State the analogous results for limit inferiors. You do not need to prove it.

2. Parts (a),(b) and (c) of problem 15 in section 2.2.

3. Parts (a),(b),(c) and (d) of problem 28 in section 2.2.

4. Problem 32 in section 2.2

5. In class we looked at the following function. Let r_n be the rationals in $(0,1)$ where $n = 1, 2, 3, \cdots$. Define

$$f(x) = \sum_{n:r_n \le x} 2^{-n}$$

We observed that this is an increasing function on $[0,1]$ and we showed that for rational x it is not continuous at x.
(a) Prove f is continuous at all irrational x.
(b) Determine with proof if f is continuous from the left and from the right at x when x is rational.