Math 565C - Spring 24 - Homework 6
Due Mon, April 29

1. Consider the 1d diffusion $dX_t = b\,dt + dB_t$ with $b \geq 0$. (So $X_t = bt + B_t$, Brownian motion with a drift.) Let $g(x) = exp(-x^2)$. As always, the problem is to study

$$g^*(x) = \sup_\tau E^x[g(X_\tau)]$$

where the sup is over stopping times τ. Find the optimal average reward $g^*(x)$ and the optimal stopping time τ^* if it exists. Note that b can be zero, and this case is qualitatively different from $b > 0$.

2. Let $X_t = B_t$, 1d Brownian motion. Let $g(t, x) = e^{-\rho t}x^2$ where $\rho > 0$. Note that the reward function depends on time. The problem is to study

$$g^*(x) = \sup_\tau E^x[g(\tau, X_\tau)]$$

Assume that the continuation region is of the form $(-x_0, x_0)$ for some x_0. Find an implicit equation that determines x_0 and find the optimal average reward $g^*(x)$ and the optimal stopping time τ^*. This problem is based on Oksendal problem 10.3, and you can find the solution (without any derivation) in Oksendal. You should provide the derivation. You do not need to prove the assumption on the form of the continuation region.

3. Oksendal problem 10.4

4. Oksendal problem 10.5