Math 565a Spring 2011 - Syllabus

This is my best guess as to what we will cover and how many lectures we will spend on each topic. This was written before the course started. There will probably be some deviations from this as the course evolves.

References

Most of the material we will cover can be found in most graduate level probability books. The three that I will use most of the time are

Watkins: Lectures notes for this course by Joe Watkins, available at his website:
http://math.arizona.edu/∼jwatkins/discretetime.pdf

Durrett: *Probability : Theory and Examples.* He put the latest edition of his book on his website. It has not yet appeared in print and may disappear from his website when it does, so I recommend you download it now.
http://www.math.duke.edu/∼rtd/PTE/pte.html

Fristedt, Gray: *A Modern Approach to Probability Theory.*

1. Review and basic definitions - 2 lectures

1.1 Conditional expectation, independence
1.2 Extension theorems
1.3 Stochastic process, filtrations, stopping times

2. Random walks - 2 1/2 lectures

2.1 Def and basics : definition of random walk, restarting a random walk at a stopping time, random number of terms.

2.2 Transience and recurrence: their definition, def of random walk on a lattice. Structure of recurrent set.

2.3 Role of dimension: conditions for recurrence in $d = 1, 2$ and for transience in $d = 3$.

Best references are Watkins - chapter 2 and Durrett - chapter 4. In FG random walks are covered back in chapter 11. This chapter includes definition of filtration and basics about filtrations. The treatment of random walks is pretty light. In particular there is not much on recurrence and transience. But note that there is a section on random walks at the end of the chapter on renewal sequences.

3. Renewal sequences - 3 1/2 lectures

3.1 Definitions and basics - definition, examples, waiting times, potential measures

3.2 Limit theorems - strong law of large numbers for renewal sequences, the renewal theorem.

3.3 Applications to random walk

Chapter 3 of Watkins and chapter 25 of FG are similar and are what we will follow. Durrent's treament of renewal sequences is split. When he does the strong law of large numbers, he also does it for renewal sequences. He has a section on renewal theory at the end of the random walk chapter. He proves the renewal theorem there.

4. Martingales - 6 lectures

4.1 Introduction : definition, examples, new martinagles from old ones.

4.2 Doob decomposition: predictable processes, the decomposition, uniform integrability.

4.3 Optional sampling theorem : conditions on stopping times, the optional sampling theorem, sufficient confitions for the stopping times. First and second Wald identities, applications (gambler's ruin, ...).

4.4 Inequalities and convergence: inequalities involving maximum of process, Doob's upcrossing lemma, submartingale convergence theorem, uniform integrability and L^1 convergence, applications

4.5 Backwards martingales : definition, backwards martingales convergence theorem, applications (strong LLN, ...).

We will follow Watkin's treatment closely. In fact, the above sections follow Watkins exactly. The martingale chapter in FG is very similar. Durrett has a chapter on martingles (chap 5). This is where he first introduces conditional expectation. He covers the topics above but in a very different order. He gets to the martingale convergence theorem first, then does the Doob decomposition. Then he does a bunch of examples including an extensive treatment of the branching process. Then he does the L^p inequalities and then uniform integrability and convergence in L^1. Next are backwards martingales. Applications include the usual strong LLN and de Finetti's theorem on exchangable sequences. His last section is on the optional sampling theorem.

5. Markov processes - 11 lectures

5.1 Definition and examples

5.2 Extensions of markov property: Chapman-Kolmogorov equation, strong Markov property,

5.3 Classificiation of states: def of recurrent, transient, period. Recurrence is contagious.

5.4 Stationary measures: def, existence and uniqueness, detailed balance, relation to return times.

5.5. Long time behavior of ergodic chains: convergence to the stationary distribution, central limit theorem for markov chains

5.6 Long time behavior for other chains

5.7 General state space: def of Markov processes for general state space, def of Harris chains, examples. Recurrence and transience, stationary measures, convergence theorem for Harris chains.

Markov processes in discrete time are simpler when the state space is countable. Durrett assumes the state space is countable for most of his chapter on Markov processes. In the last section he consider uncountable state spaces, in particular Harris chains. Watkins starts with a general state space and proves some things in that setting, e.g., the strong Markov property. But he restricts to a countable state space when he gets to classification of states and convergence results. FG has a chapter on Markov processes. While it talks about stationarity distributions, it does not study convergence to stationary distributions - a fatal flaw. I will probably follow Durrett most closely.

6. Stationary processes - 5 lectures

6.2 Basics : definition of one-sided and two sided invariant sequences, σ-field of invariant events, examples.

6.2 Birkhoff's ergodic thm: maximal ergodic lemma, proof of ergodic thm.

6.3 Ergodicity and mixing: def of weak and strong mixing, relation to ergodicity.

6.4 Subadditive ergodic thm

The above topics follow FG most closely. They also have a section on spectral theory for stationary sequences. Watkins does not cover the subadditive ergodic theorem, but has a section on entropy. Durrett includes some discusson of recurrence.