Monte Carlo Methods - Spring 16 - Homework 3

1. (antithetic) Consider the network example.

(a) In class and in the notes I used the antithetic random variable $Y = f(1-U)$ where $1-U$ means $(1-U_1, 1-U_2, 1-U_3, 1-U_4, 1-U_5)$. I find that the variance is reduce from about 0.1577 for MC without the antithetic variable to 0.01823 with the antithetic RV. Reproduce this result.

(b) Now let $\tilde{U} = (1-U_1, U_2, U_3, 1-U_4, U_5)$ and let $Y = f(\tilde{U})$. Note that $\tilde{U}$ has the same joint distribution as U, so Y is still a valid antithetic RV. How much variance reduction do you get? Can you explain this? What happens if I do some other tranformation of the probability space, e.g.,
$\tilde{U} = (U_1, 1-U_2, 1-U_3, U_4, U_5)$?

2. (control variates) In the network example in class I took $Y = T_1 + T_4$ and then just set $\alpha = 1$ so that the RV we do MC on is just $Z = X - (Y-1)$. I find that the variance is reduced from about 0.1577 for MC without a control variate to 0.0412 with this control variate. Now let $Z = X - \alpha(Y-1)$. What is the optimal α and how much variance reduction do you get?

3. (control variates) From Fishman's *First course...* We want to compute the following integral by Monte Carlo:

$$\int_0^1 e^{-x\cos(\pi x)}\, dx = E[f(U)]$$

where U is uniform on $[0,1]$ and $f(x) = e^{-x\cos(\pi x)}$. Suppose we use the control variate $Y = g(U)$ with $g(x) = e^{-x}$. Note that we can compute the mean of Y explicitly. By comparing a MC simulation with and without this control variate (using $\alpha = 1$) find the variance reduction from the control variate. Fishman find it is a factor of about 2.9.

4. (partitioning) Consider the rainfall example from Owen's book, section 8.5, chapter 8. In class I looked at the varaince reduction from using proportional allocation (following Owen very closely).

(a). Reproduce this result.

(b). Now suppose we choose the sample sizes to be optimal. So n_j is proportional to $p_j\sigma_j$ where p_j is the probability of the jth strata and σ_j^2 is the variance within the jth strata. Normally you would do a preliminary MC to estimate the σ_j. If you want to skip this you can find estimates of their

values in Owen. With this optimal choice of the number of samples what is the variance reduction?

5. (partitioning) Consider the network example. There is no obvious way to partition. Come up with some partition and see how much variance reduction it gives. To begin with you can use proportional allocation to choose the number of samples in each strata. If you are more ambitious you can compute the variance within each strata and use the optimal sample sizes for the strata.