Monte Carlo Methods - Special Topics Course

Spring 2016, Instructor: Tom Kennedy

Tentative syllabus

Meeting time: Mon/Wed 1:00-2:15

This will be a graduate level course on Monte Carlo methods with a particular emphasis on Markov Chain Monte Carlo (MCMC). Monte Carlo methods generate random samples in order to compute a quantity of interest. The quantity of interest need not be random. With modern computing power Monte Carlo methods have become a powerful tool used in virtually every field of science and engineering. Try googling "markov chain monte carlo ?" where ? is biology, physics, machine learning, optical science, or your favorite area of science or engineering, and see what you get.

Topics:

1. Direct (or simple) Monte Carlo
   (a) generating uniform random numbers
   (b) generating continuous and discrete random variables,
   (c) generating multidimensional Gaussian processes (Cholesky),
   (d) rejection sampling
   (e) importance sampling
   (f) review of CLT and confidence intervals as background for next topic
   (g) error estimates for simple monte carlo
   (h) statistical tests relevant to MC, e.g., Kolmogorov-Smirnov goodness-of-fit
2. Markov chain Monte Carlo (MCMC)
   (a) the requisite Markov chain theory and the key idea of MCMC
   (b) the Gibbs sampler
   (c) the Metropolis-Hasting algorithm
   (d) autocorrelation times and error bars
   (e) error bars via batched means
   (f) burn-in time
   (g) examples from various fields including Bayesian statistics
3. Further topics:

(a) variance reduction
(b) sequential monte carlo
(c) adaptive monte carlo
(d) simulated annealing
(e) stratified sampling
(f) perfect sampling
(g) optimizing functions evaluated by Monte Carlo
(h) rare event simulation
(i) other topics to be chosen based on the students interests

Prerequisites: This course is aimed at PhD students in Mathematics, Applied Mathematics, Statistics and any field where Monte Carlo methods are used. I will assume probability at an advanced undergraduate level, up to and including the Law of Large Numbers and the Central Limit Theorem, basic linear algebra and differential equations at an advanced undergraduate level, and basic scientific computing literacy and the ability to program in a language suitable for scientific computing. Some familiarity with Markov chains is helpful, but I will do a quick treatment of what we need.

Requirements : A course project. It is expected that for many students in the course their research will involve Monte Carlo methods of some form. Projects taken from the students' research are certainly acceptable, even encouraged. There will also be some problem sets, but they will be optional.