Linear Algebra – Check your understanding

□ Is it possible for a linear system of equations to have exactly 10 solutions? Why or why not?

□ Is it possible for a linear system of equations to have no solution at all? If so, give an example. If not, explain why.

□ Is the determinant defined for a matrix that is not a square matrix?

□ Why is it useful to manipulate a determinant in order to changes some of its entries into zeros?

□ Can you explain why the determinant of a matrix is equal to zero if its columns are linearly dependent?

□ Conversely, if the determinant of a matrix is equal to zero, does it mean that the columns of the matrix are linearly dependent? Why or why not?

□ Is it possible for an eigenvector to be zero?

□ Is it possible for an eigenvalue to be zero?

□ If A is not a square matrix, is it possible to find eigenvalues of A? Why or why not?

□ When you look for eigenvectors of a matrix A associated to a particular eigenvalue λ, do you expect to find exactly one eigenvector, or is there an infinite number of possible eigenvectors? Explain.

□ Give an example of a 3 by 3 matrix whose rank is 1. What is the dimension of the null space of the matrix you just found? Explain.

□ Give an example of three 3-dimensional vectors that do not span $\mathbb{R}^3$. Choose the vectors so that no two vectors are proportional to one another.

□ Give an example of three 3-dimensional vectors **with non-zero entries** that span $\mathbb{R}^3$.

□ List three different ways of finding the rank of a matrix. Explain why the 3 methods give the same answer.