Response to Mochizuki's comments on my papers

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I address Shinichi Mochizuki's recent comments on my work. This document was emailed to Mochizuki ten days ago for his comments. The boxed expressions in red are references to specific points in Mochizuki's document.

(1) [RC-Ex1]: Let me quote what you say in your paper regarding arithmetic holomorphic structures [Mochizuki, 2021a, Page 20]:

> *"one natural and fundamental problem, which will, in fact, be one of the main themes of the present series of papers ([Mochizuki, 2021a,b,c,d]), is the problem of describing an alien "arithmetic holomorphic structure" [i.e., an alien "conventional scheme theory"] corresponding to some Hodge Theater HT_n in terms of a "known arithmetic holomorphic structure" corresponding to HT_m [where $n \neq m$] $\cdots$"*

Clearly a plurality of Arithmetic Holomorphic Structures are required for your proof and hence must be exhibited. For readers unfamiliar with the technical term 'Hodge Theater' and its role of a Hodge Theater in [Mochizuki, 2021a,b,c,d] one has the following quote from [Mochizuki, 2021a, Page 20]

> *"In the context of the above discussion, we remark that the word "Hodge" in the term "Hodge theater" was intended as a reference to the use of the word "Hodge" in such classical terminology as "variation of Hodge structure" . . . That is to say, later, in [IUT III], we shall see that the position occupied by a "Hodge theater" within a much larger framework that will be referred to as the "log-theta-lattice" [cf. the discussion of [Mochizuki, 2021a, § I4 below] corresponds precisely to the specification of a particular arithmetic holomorphic structure in a situation in which such arithmetic holomorphic structures are subject to deformation."*

To put it transparently: the collection of Hodge Theaters in your "log-theta-lattice" is a proxy for a variation of (p-adic) Hodge structures. Now the subject of p-adic Hodge Theory has undergone a dramatic transformation in the past two decades. As you remarked to me in June 2021 (Kyoto Conference SLACK/Zoom) and again in your recent comments cited above, you have not studied this modern avatar of p-adic Hodge Theory–because if you had, then you would have agreed with my precise mathematical conclusions (which provide robust proofs of your assertions).

In late 2019/early 2020, I discovered, quite serendipitously, that my knowledge of modern p-adic Hodge Theory could be applied effectively to the problem (arising in the context of your papers) of describing the canonical plurality of arithmetic holomorphic structures (and hence, to put it in your terms, describe a canonical plurality of Hodge Theaters). That is why the theory I construct matches so accurately and precisely with the theory you have claimed in your papers.

Now let me discuss the example of projective line "$T = T^{-1}$" which you have repeatedly cited as a means of understanding your theory [Mochizuki, 2021a,b,c,d]. I think that this example is not useful in persuading anyone of any theory purporting to prove a subtle and deep conjecture such as the abc-conjecture. I also think that the projective line example is context-free because, in the case of the projective line, a new geometric object is created by this identification ("$T = T^{-1}$"). On the other hand, in the context of Hodge-Theaters no new object is created and each of the Hodge-Theaters, HT_n for $n \in \mathbb{Z}$, make a separate contribution to the theta-values locus which enters [Mochizuki, 2021c, Corollary 3.12]. For a precise version of the gluing assertion see [Joshi, 2023b, Theorem 10.10.1, 10.11.5.1].

(2) [RC-Ex2]: the precise assertion which needs to be proved here is [Joshi, 2023a, Theorem 5.6.1].

(3) [RC-Ex3]: as you know well, in classical Teichmuller Theory, it is possible for two distinct points of a Teichmuller space of a Riemann surface to provide conformally equivalent Riemann surfaces. The two points may be distinguished by means of Teichmuller's equivalence of quasi-conformal mappings.

In your work, you have avoided any discussion of Berkovich analytic structures because they cannot be reconstructed by your Anabelian Reconstruction Theory, and instead you have attempted to create a purely group theoretic notion of arithmetic holomorphic structures.

I recognized early on (and proved) that in fact working with Berkovich analytic structures brings substantial clarity to the situation considered in [Mochizuki, 2021a,b,c,d] and provides a more natural definition of arithmetic holomorphic structures which comes equipped with a natural variation of p-adic Hodge structures (see **(1)** above). My approach provides a natural definition of quasi-conformal equivalence in the p-adic context and in particular allows one to explicitly demonstrate that p-adic metrics exhibit the sort of stretching and scaling behavior seen in complex quasi-conformal mappings.

(4) The global aspects of my theory are considered in [Joshi, 2023a]. Since there are no references to it in your comments, it is quite likely that you have not looked at it.

(5) I will address local/global concerns in a separate document and update my papers [Constr. III, IV] accordingly.

(6) As my LaTeX source files of my preprints on arxiv.org will testify, I use the standard automated theorem numbering provided by AMSLaTeX (American Math. Society, LaTeX).

Any numerical coincidences anyone finds in the theorem numbering in my papers must be considered entirely self-imagined.

(7) Besides your public comments (mentioned above) on my work, I have received no email communications from you or from any other IUT experts even though I have sent copies of all my works on these topics over the past few years.

Summary

All in all, I have believed, and asserted (in all my papers on this topic) that you have presented rather new ideas in Diophantine Geometry and I have shown that these ideas can be made precise using a new set of tools (especially my use of perfectoid fields and untilts in this context) which are better suited for this purpose than the ones you have created. So if there is a proof of the abc-conjecture in [Mochizuki, 2021a,b,c,d] (and I certainly *do* think so), my work will also arrive at this conclusion–after possible missteps (I am not worried about being wrong–the act of doing mathematics is not without pitfalls, but ultimately it is a self-corrective cerebral process).

Therefore, in the interest of the underlying mathematical results, I urge you to be open minded about my work and I hope you will be open to discussing your results with me and help settle this matter with greater clarity once and for all.

References

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