Diffusion: from experiments to geometry

In 2008, in collaboration with an experimental physicist, Giovanni Volpe, I started a project on Brownian motion. The main problem was to explain emergence of a physically measurable quantity, which we later named {\it noise-induced drift}. The theory we developed for this purpose turned out to have a wide range of applications and led to several new research projects, ranging from further experimental work aimed at physical applications, to mathematical questions about diffusion processes on Riemannian manifolds. In my two lectures I am planning to give a short introduction to the subject, focusing on open mathematical questions and possible dissertation problems.