Abstract: Most 3d rendering algorithms represent surfaces as collections of triangular patches, i.e., use 1st order approximation. While this representation allows for very fast rasterization, it requires a lot of triangles to model smooth surfaces. The calculations become very challenging even for modern hardware as soon as one wants to incorporate physically-based lighting, as their computation complexity scales polynomially with the number of primitives (triangles). One way to address this issue is to use higher order surface approximation. In particular, quadratic (2nd order) approximation seems to be a very attractive choice due to its small memory footprint and possibility of finding the ray-surface intersection points exactly. It turns out that there are still plenty of unsolved mathematical problems in the theory of quadratic spline interpolation, however, that impede progress in this direction. I will discuss the history of this subject, some open problems and ideas for addressing them.