

# TOPICS IN THE GEOMETRY OF GROUPS

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A very fruitful idea in the geometric group theory is the introduction of the *word length metric*: let  $S$  be a symmetric system of generators of a group  $G$ , define a metric  $\rho$  on  $G$  by letting  $\rho(g, g') = \#$  of generators in  $S$  needed to write  $g^{-1}g'$ . The group and its Cayley graph become naturally metric spaces and one can study questions similar with those in Riemannian geometry: curvature, geodesics, Laplacian etc. The goal of this course is to understand by way of examples some of this geometry of groups, together with the structure ‘at infinity’ that can be carried by a metric space.

The course should be of interest to students that plan to work in geometry or analysis. There are really no prerequisites. We shall cover all the details in class. Due to the combinatorial aspect of the theory, the class may be accessible to advanced undergraduates.

Topics will include:

- (1) Free groups and free product of groups
- (2) Word length and Cayley graphs
- (3) Quasi-isometries
- (4) Fundamental groups and curvature in Riemannian geometry
- (5) Gromov’s hyperbolic groups and boundaries
- (6) Amenability
- (7) Polynomial and exponential growth
- (8) Group actions on trees
- (9) Finite asymptotic dimension
- (10) Property A, embeddability in Hilbert space, expanders (time permitting)
- (11) John Roe’s work on coarse geometry (time permitting)

The text book will be [1]. The rest of the titles below are interesting and relevant to the subject.

## REFERENCES

- [1] Pierre de La Harpe, *Topics in geometric group theory*, University of Chicago Press, 2000.
- [2] Martin R. Bridson and Andre Haefliger, *Metric Spaces of Non-Positive Curvature*, Springer Verlag, 1999.
- [3] Etienne Ghys and Pierre de la Harpe, *Sur les groupes hyperboliques, d’apres Mikhael Gromov*.