

## Special Topics Course: Discrete Dynamical Systems and their Applications

Discrete Dynamical Systems have often been used as a means to explore or better understand more complicated mathematical and physical settings involving ordinary or partial differential equations. Moreover, because of the comparative simplicity of their basic structure these systems enable connections to other areas of mathematics, both as applications and tools for study, and they are more readily suitable for direct numerical simulation.

A classical example of this was the study of iterated one-dimensional maps by Feigenbaum which revealed aspects of the nature of chaotic dynamics (and its hidden internal order) that subsequently could be extended to many much more complicated physical models in fluid dynamics and optical systems. Feigenbaum's studies were easy to explain in elementary terms (a bright high school student could grasp what he was doing immediately) and the results were strikingly visualizable, intuitively engaging and naturally stimulated curiosity and a desire to explore further.

A renewed interest in discrete systems has emerged in recent years through cross-fertilization between different areas of mathematics; some of that taking place here at Arizona, especially in relation to random matrix theory and integrable systems theory. A sampling of just a few of these topics is:

- Tropical Mathematics
- Laser Models
- Billiards (or area preserving maps of planar domains)
- Random Surfaces
- Random Schrodinger equations

The basic plan for the course will be to start out with one-dimensional maps and the Feigenbaum analysis as a means to introduce the general topic. We will then move on to two-dimensional systems illustrated by billiard systems and the Ikeda map and culminating with a description of a general method for describing those maps which are completely integrable. In all of this we expect to employ both numerical and analytical tools. The remainder of the course will sample applications from the above list and perhaps other areas depending on student interests. Anyone interested in a more detailed description of the above listed topics can email me (ercolani@math.arizona.edu) and these descriptions will be provided.

*Prerequisites:* A good knowledge of Differential Equations or Dynamical Systems Theory at the upper division level at least (such as Math 454) OR one of the graduate core courses in Analysis (Math 523 or Math 527).