Topics on Moduli Problems.

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Overview: Moduli problems are of central importance in algebraic geometry. It has deep connections to other branches of mathematics such as symplectic and differential geometry. In the last few decades, it also interacts deeply with theoretical physics. Among many moduli spaces, the following are two important types: A. moduli of varieties (of a fixed type); B. moduli of vector bundles (sheaves) over a fixed variety. Moduli of type A that are well studied include: moduli of curves of genus g, moduli of K3 surfaces, etc, Moduli of type B that are well studied include: moduli of vector bundles over a curve of genus g, moduli of sheaves over a surface. The aim of this course is to introduce these moduli spaces and their foundations to students.

Books：

1. Geometric Invariant Theory, by Mumford et al

2. Moduli of Curves, by Harris and Morrison

3. Deformation Theory, By Hartshorne

Description: In this topics course, I will cover the following fundamental concepts/tools in moduli problems in algebraic geometry.

1. Geometric Invariant Theory.
2. Moduli Spaces and Moduli Stacks.
3. Examples of Moduli Problems, such as moduli of curves and stable maps.
4. Deformation Theory

Prerequisites: The course should be accessible to anyone who has taken one-semester Math 536 or the equivalent. It is also expected that the students have already taken graduate courses on algebraic topology and differential geometry.

Expected Learning Outcomes:

Be able to describe some moduli spaces and their basic properties;

Be able to present the basic ideas in geometric invariant theory (GIT) and point out some example of moduli spaces that have been successfully constructed by GIT.

Be able to describe the basic ideas of moduli stacks.