Topics in Discrete Probability

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Overview: In this course, we consider two themes in discrete probability, which serve as bases for current research:

(1) Random walks and electrical networks
(2) Statistical physics: Percolation and Ising models

Description and references: In the first part, we will cover material from books *Markov chains and mixing times* by Levin, Peres and Wilmer (Ch 9, 21), as well as from *Random walks and heat kernels on graphs* by Barlow (Ch 2). This material is also more informally presented in monograph *Random walks and electrical networks* by Doyle and Snell.

The idea is that some behaviors of random walk on graphs, such as recurrence/transience, escape probabilities from sets, etc., can be captured by considering the conductance, electrical current, capacity, voltage, and resistance structure of the graph. In some sense, this relates notions of `potential theory' to `random walks'.

The second part discusses percolation and Ising models on certain graphs/lattices, which relate to studies of disorder in various media. In percolation, a quantity of interest modeled is the chance that a drop of water can pass through `cracks' in a material. In Ising models, one studies notions of magnetism, etc. The concept of phase transition will be introduced. Relevant sources will be lecture notes of Steif, *Probability: the classical limit theorems* by McKean, *Percolation* by Grimmett, a recent book *Statistical mechanics of lattice systems* by Friedli and Velanik, and some notes that I have, etc.

Prerequisites: There are minimal prerequisites—a previous course in probability, some background with Markov chains will be useful, but things also will be introduced as we go along.

Approximate syllabus: After an introduction to Markov chains, in the first week, the next 4 weeks will focus on the random walk on graphs and connections to electrical networks. The next 3-4 weeks will be on percolation and the remaining 3-4 weeks on the Ising model. Possible research directions will be discussed in the last week.

Expected learning outcomes: After the course, participants will be able to calculate precisely behaviors of random walks in terms of electrical network notions. Also, participants will be able to state properties of models of disordered systems such as percolation and the Ising model in a mathematical way. With respect to both parts, participants will be able to state theorems underlying the phenomena along with their proofs.

Questions are welcome: sethuram@math.arizona.edu