

Title: Regularity of solutions for nonlinear PDE

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Text: Self-generated notes

Description of topics to be covered: Classical techniques (De Giorgi-Nash-Moser iteration, Schauder estimates) for obtaining regularity of PDE, as well as modern ones (Krylov-Safonov) and their application to well-posedness problems and qualitative understanding of solutions to PDE arising in applications. I plan to touch on several related topics such as Hamilton-Jacobi equations, viscosity solutions, homogenization, etc.

Prerequisites: Knowledge of Sobolev spaces

Learning outcomes: Successful students should understand regularity issues with weak solutions, identify cases where higher regularity is expected, and be able to apply the major theorems and techniques to deduce such regularity. The successful student should understand the connection between the establishment of regularity estimates and the analysis of qualitative behavior of PDE (e.g. long-time behavior, homogenized approximate models, etc.).

Approximate schedule:

- * Weeks 1-5: Hilbert's 19th problem and its resolution by De Giorgi and Nash
- * Weeks 6-7: Applications of regularity estimates to biological models (e.g. nonlocal Fisher-KPP, Keller-Segel)
- * Weeks 8-13: Nonlinear PDE, viscosity solutions, Krylov-Safonov Theorem
- * Week 14: Applications of Krylov-Safonov to obtain "surprising" regularity results (e.g. Hamilton-Jacobi equations, kinetic equations)
- * Remaining time: topics in homogenization