Introduction to p-adic Hodge theory

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Course description: P-adic Hodge theory is the study of p-adic representations of the Galois group of a p-adic field arising out of the study of arithmetic geometry over p-adic fields. Since its development beginning in 60-70s in the pioneering work of Jean-Marc Fontaine, it has played a important role in many significant developments in number theory including the proof of Fermat's Last Theorem, the Mordell conjecture, and Serre's conjecture for modular forms to name just a few. The field continues to develop rapidly following Peter Scholze's introduction of perfectoid spaces and new p-adic cohomology theories. The goal of this course is to cover Fontaine's theory of period rings and the fundamental theorems of p-adic Hodge theory. In the second half of the course, we will then proceed to integral theories classifying lattices in p-adic Galois representations. Finally, we will end with a survey of some of the recent developments like perfectoid techniques.

Prerequisites: Galois theory of local fields as covered in Math 514A,B and Algebraic geometry equivalent to Hartshorne or 536A

References: Conrad and Brinon's Notes from CMI summer school (http://math.stanford.edu/~conrad/papers/notes.pdf) and Berger's "An Introduction ot the theory of p-adic representations" (http://perso.ens-lyon.fr/laurent.berger/articles/article05.pdf)

Learning outcomes:

1. A working knowledge of the main techniques/results in p-adic Hodge theory
2. Familiarity with the semilinear algebra objects appearing in the theory
3. An understanding of the connections to important problems in algebraic number theory

Schedule:

Weeks 1-4: Introduction to Fontaine's theory, Hodge-Tate and De Rham theory

Weeks 5-8: Finite flat groups schemes and crystalline theory

Weeks 9-12: Integral theory

Weeks 13-16: Recent developments