

Title: p-adic analysis and zeta functions

C. Douglas Haessig

Description: The course will be an introduction to the study of L-functions of exponential sums and zeta functions of algebraic varieties over finite fields using p-adic analysis. The course will be broken up into the following sections:

1. Basics of p-adic numbers and p-adic analysis
2. Intro to L-functions and zeta functions
3. Dwork's trace formula and Rationality
4. p-adic cohomology theory

Texts:

- 1) Koblitz "p-adic numbers, p-adic analysis, and zeta functions"
- 2) Monsky "p-adic analysis and zeta functions"
- 3) Various papers in the literature

Prereq: Course will be kept mostly self-contained. Undergrad knowledge in algebra (some Galois but not much), linear algebra, analysis.

Prerequisites:

Math 511A or instructor consent.

Learning Outcomes:

- a) Basic understanding of p-adic numbers and p-adic analysis and how it may be used to lift questions from finite fields to characteristic 0.
- b) Exposure to the Weil Conjectures on zeta functions, with a proof of the first conjecture, rationality of the zeta function.
- c) An introduction to Dwork's pioneering work, including his p-adic analytic lifting of an additive character over a finite field, and his trace formula, which relates the trace of Frobenius with exponential sums.
- d) Introduction to p-adic cohomology theory (de Rham type) in a self-contained way.

Tentative Schedule

Introduction to p-adic numbers (ie. Ultra normed fields)

Weeks 1 - 3Covers:

Koblitz book: Chapters 1, 3

Dwork G-function: Chapt 1, sections 1 - 9

Introduction to p-adic analysis

Weeks 4 - 6Covers:

Koblitz book: Chapter 4, 5

Dwork G-function booklet: Chapt 2, sections 1 - 5

Introduction to zeta functions and L-functions

Weeks 7 - 10 Covers:

Koblitz book: Chapt 6

Dwork G-function book: Chapt 2, sections 6 - 9

Introduction to cohomological methods

Weeks 11 - 15

Covers: Koszul complexes; p-adic cohomology

Amalgam of various papers in the literature

Dwork's Bessel paper

Adolphson-Sperber's Annals paper

Monsky book: bits of chapters 6, 7, 8