Title: Inference, Learning and Optimization with **Graphs and Networks**

Instructor: Michael Chertkov

Proposed as a Special Topic Math577 course for the Fall of 2022

Description: This introductory course is suitable for graduate students in both the mathematical sciences (Mathematics, Applied Mathematics, Statistics / Data Science) as well as in computer science, school of information, engineering, physical, social and biological sciences, and economics. The course is aimed at the students interested in learning about modern theoretical and practical approaches, which are in the core of modern modeling developments in Artificial Intelligence and Machine Learning for analysis of big data sets with reach statistical and deterministic correlations, constraints and relations expressed through graphs, networks, matrices, tensors and related. In this course you will learn theoretical concepts, will develop mathematical and common sense intuition, and will also become familiar with many modern applications of the graphical and network concepts in sciences and engineering.

Prerequisites: Knowledge of the basic undegraduate mathematics (algebra, analysis, differential equations) is assumed/required. Some prior experience in Applied Mathematics, Probability Theory, Statistics, Statistical Mechanics or Machine Learning (at least one graduate level course) is recommended.

Assignments & Credits: There will be two graded home-works and an individual project. No midterms, no finals. Grade structure: Lecture Attendance 10%; HWs 40%; Project 50%. List of projects (and related research papers) will be given (30+ options) and student-suggested projects are encouraged. Students are required to pick up one of the suggested projects or suggest their own paper/project relevant to the course (negotiable) by Sep 15. All projects will include software implementation component. Any modern scientific software will be acceptable for completing the projects, including matlab, mathematica, julia and python with the preference to the later one. Project presentations will be scheduled (possibly multiple sessions) for November-December.

Textbooks: No required textbook. Extensive course-notes (200 pages) and course-linked software (in python on github) will be provided. A number of books and other online resources will be recommended including:
1. Graphical models, exponential families, and variational inference, Foundations and Trends in Machine Learning, Martin Wainwright & Michael Jordan, ISBN-13-978-1601981844.
2. Probabilistic Graphical Models: Principles and Techniques, Daphne Koller and Nir Friedman, ISBN-13-978-0262013192.
3. The Nature of Computation, Cris Moore & Stephan Mertens, Oxford 2011, ISBN-13-978-0198570837.
4. Information, Physics and Computation, Marc Mezard & Andrea Montanari, Oxford 2009, ISBN-13-978-0198570837.
5. Optimization Algorithms in Physics, Alexander K. Hartmann & Heiko Rieger, Wiley-VCH, 2002, ISBN-13-978-3527403073.
6. Reinforcement Learning: An Introduction, Richard S. Sutton & Andrew G. Barto, ISBN-13-978-0262039246.

Expected learning outcomes: After completion of the course students will
• Learn how to formulate graphical model and how to do related computations. • Know when (applications) and how (methods and algorithms) to use Graphical Models and Networks for inference and learning and optimization.
• Understand general methodology of Graphical Models and Networks for representing probability distri-

butions in high dimensions, related algorithms and computational tools. • Acquire study and presentation skills (oral and written).

Topic	Summary of Topic
Graphical Models - setting the stage, motivations, applications	Structured Statistical Inference (problem formulations – sampling, marginal probabilities, partition functions) in Computer Science, Information Theory and Physics (intro)
Computational Complexity & Algorithms	Deterministic & Stochastic approaches. Statistical Inference as an Optimization – from Partition Function and Marginal Probabilities to Free Energy (Kublack-Leibler Functional). i.i.d. sampling.
Triage of methods for approximate inference	Variational (optimization) Approaches: Mean-Field, Belief Propagation, Linear Programming, Relaxations, Lower and Upper Bounds. Exact & Heuristic approaches. Iterative and Distributed Algorithms. Variable Elimination Approaches: Dynamic Programming (exact and approximate), Variable Elimination, Mini-backet Elimination, Tensor Networks. Stochastic Approaches: MCMC, FPRAS. Synthesis (mixing) of the approaches.
Modern Analysis and Algorithmic Tools	Review of Advanced inference and optimization methods: Loop Series, Cummulant Expansions, Computational Trees, Graph Cover & Monte-Carlo Approaches, convex relaxations, upper and lower bounds.
Neural Networks as Graphical Models	What is easy (inference) and what is difficult (learning) in Neural Networks. Chow-Liu (tree) Learning. Learning of Graphical Models and Neural Networks as an Optimization (convex and not convex). Restricted Boltzmann Machines. Learning under Partial Observability. Expectation Maximization.
Graphical Models and Networks for Dynamical Systems	Directed (e.g., Acyclic) Graphical Models. Causality. Graph and Network approaches to Markov Processes, Markov Decision Processes and Reinforcement Learning. Graph and Network Modeling with Agents.
Real World Examples	Depending on the interest of the attendees there will be some additional theoretical and application topics covered in the lecture. Possible Additional Theoretical Topics: (a) Network Flows; (b) Attractive (Ferromagnetic) Ising Models; (c) Matching Models; (d) Planar (det-reducable) Models; (e) Gaussian Graphical Models. Possible Application Topics: (a) Fluid Mechanics; (b) Energy Systems; (c) Epidemiology; (d) Economics. We will discuss connections/links to other areas of research in contemporary applied mathematics, modern theoretical engineering and related. Open problems/challenges.