Do either one of $nA$ or $nB$ for $1 \leq n \leq 5$. Justify all your answers.

1A. Let $A$ be a complex 6 by 6 matrix. Suppose that $A^3 = I$. List the possible Jordan canonical forms for $A$.

1B. Find a real orthogonal 2 by 2 matrix $P$ such that $P^{-1}AP$ is diagonal for

$$A = \begin{pmatrix} 6 & -2 \\ -2 & 3 \end{pmatrix}.$$

2A. Let $G$ be a finite group. Let $p$ be the smallest prime dividing the order of $G$. Show that any subgroup of $G$ of index $p$ is normal.

2B. Show that the group given by the presentation $\langle a, b | a^2, b^2 \rangle$ is an infinite group.

3A. Recall that a ring $A$ is artinian if every descending chain of ideals stabilizes. Let $A$ be commutative ring with 1 that is artinian and an integral domain. Prove that $A$ is a field.

3B. Find a maximal ideal in $\mathbb{C}[x, y]$ that does not contain $xy$ and find a prime ideal that is not maximal and does not contain $xy$.

4A. Let $k$ be a field. Answer true or false for the following statements. If true, then very briefly sketch a proof outline (1-3 lines). If false, then state an explicit counterexample.

(1) Every field extension of $k$ of degree 2 is normal.

(2) Every field extension of $k$ of degree 2 is of the form $k(\sqrt{\beta})$, where $\beta \in k$. 
4B. Determine the Galois group of the polynomial \( x^3 - 2 \)
   a) over \( \mathbb{Q} \),
   b) over \( \mathbb{F}_7 \),
   c) over \( \mathbb{F}_9 \).

5A. Let \( R \) be a commutative ring with 1. Let \( M \) and \( N \) be finitely generated \( R \)-modules. Prove that the tensor product \( M \otimes_R N \) is a finitely generated \( R \)-module.

5B. Let \( R \) be a finite semisimple ring with 1. Suppose that no fourth power \( n^4 \) for \( n \in \mathbb{N}, n > 1 \), divides \( |R| \). Show that \( R \) is commutative.