The minimal polynomial is
\[
\begin{pmatrix}
0 & a_0 \\
1 & 0 & : \\
& 1 & \ddots & : \\
& & \ddots & 0 & a_{n-2} \\
& & & 1 & a_{n-1}
\end{pmatrix}
\]
Here \(a_0, \ldots, a_{n-1}\) are complex numbers.

1B. Find the determinant of the following rational \(n\) by \(n\) matrix for \(n \geq 3\).

\[
\begin{pmatrix}
2 & -1 & 0 & \cdots & 0 \\
-1 & 2 & -1 & \ddots & : \\
& 0 & \ddots & \ddots & \ddots & 0 \\
& & \ddots & -1 & 2 & -1 \\
0 & \cdots & 0 & -2 & 2
\end{pmatrix}
\]

2A. Determine the group of automorphisms of the symmetric group \(S_3\).

2B. Show that the symmetric group \(S_4\) is isomorphic to \(\text{GL}_2(\mathbb{F}_3)/\text{Z}(\text{GL}_2(\mathbb{F}_3))\), where \(\mathbb{F}_3\) is the field with three elements.

3A. Let \(A\) be a unital commutative ring and \(P\) be a prime ideal of \(A\). If \(I_1, \ldots, I_n\) are ideals of \(A\) and \(P = \bigcap_{i=1}^n I_i\), then \(P = I_i\) for some \(i\).

3B. (1) Prove the following statement: If \(R\) is a commutative ring, then the set of nilpotent elements of \(R\) is an ideal of \(R\).

(2) Prove or disprove: if \(R\) is an arbitrary ring then the set of nilpotent elements of \(R\) is an ideal of \(R\).

4A. Determine the Galois group of \(x^n - t \in \mathbb{C}(t)[x]\) over \(\mathbb{C}(t)\).

4B. Let \(F\) be a field of characteristic \(p > 0\) and \(a \in F\). Consider \(f(x) = x^p - x - a\). Assume that \(f(x)\) is irreducible, determine the Galois group of \(f(x)\).

5A. Find (up to isomorphism) all semisimple rings with 324 elements.

5B. Let \(A\) be a unital commutative ring and \(M\) be a noetherian module over \(A\). Assume the following condition: if \(a \in A\) and \(am = 0\) for all \(m \in M\), then \(a = 0\). Show that \(A\) is a noetherian ring.