# Geometry-Topology Qualifying Exam

## Spring 2014

**Problem 1**
Evaluate the integral

$$\int_{-\infty}^{+\infty} \frac{x}{1+x^4} e^{-ipx} dx, \qquad p > 0.$$

**Problem 2**
Let $S = \{x^2 + y^2 + z^2 = 9\}, H = \{x^2 + y^2 = z^2 + 1\}$, and $X = S \cap H$.
a) Is $X$ compact? Is $X$ connected? For some $x_0 \in X$ what is $\pi_1(X, x_0)$?
b) Prove that $X$ is a regular submanifold of $\mathbb{R}^3$.

**Problem 3**
Define

$$\begin{aligned} f &: \mathbb{R}^2 \to \mathbb{R}^2 \\ (x, y) &\mapsto (u, v) = (3x^2 + y, 5xy), \end{aligned}$$

and let $\omega = du \wedge dv$. Evaluate

$$f^*(i_V \omega),$$

where

$$V = u\frac{\partial}{\partial u} + v\frac{\partial}{\partial v}.$$

**Problem 4**
a) Explain why the map

$$\begin{aligned} f : \mathbb{C} \setminus \{0, \pm 1, \pm i\} &\to \mathbb{C} \setminus \{0, 1\} \\ x &\mapsto w = z^4, \end{aligned}$$

is a normal (or regular) covering space.
b) Find a set of generators for the subgroup of $\pi_1 (\mathbb{C} \setminus \{0, 1\}, 1/2)$ which corresponds to this covering.

**Problem 5**

a) Find a CW complex for $\mathbb{R}P^3$.

b) Use this CW complex to compute the Euler characteristic and homology of $\mathbb{R}P^3$.

**Problem 6**

For each of the following statements, either briefly explain why the statement is true or give a counterexample.

a) Every exact $k$-form on a compact orientable $k$-dimensional manifold vanishes at some point.

b) Suppose that $f : X \to Y$ is a smooth mapping of smooth manifolds. If $f$ is one-to-one and onto, then $f$ is a diffeomorphism.

c) If the degree of a smooth map $f : S^2 \to S^2$ is nonzero, then the map $f$ is onto.