Integration workshop 2020
– Single Variable Calculus –

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1 Problems: Taylor Series

1. Determine which differentiability class each of the functions belong to:

   (a) $f(x) = \frac{1}{1-x}$ for (i) $-1 < x < 1$, and (ii) $x \in \mathbb{R}$.

   (b) $f(x) = x\,|x|$ for (i) $1 < x < \infty$, and (ii) $x \in \mathbb{R}$.

   (c) $f(x) = x^{4/3}$ for (i) $1 < x < \infty$, and (ii) $x \in \mathbb{R}$.

2. Derive (or state) the first three non-zero terms of the Taylor Series centered at 0 for the following functions:

   (a) $\ln(1+x)$

   (b) $\cos(t^2)$

   (c) $\operatorname{sinc}(kx) := \begin{cases} \frac{\sin(kx)}{kx} & x \neq 0 \\ 1 & x = 0 \end{cases}$

   (d) $(1+x)^\alpha$ for $\alpha > 0$

3. Describe how you could use a Taylor polynomial to estimate the integral $\int_{-1}^{1} e^{-x^2}\,dx$ to 2 decimal places.

4. Identify the domains on which the following functions are (real) analytic.

   (a) $f(x) = 1 - 3x + 4x^3$

   (b) $f(x) = e^x$

   (c) $f(x) = (1+x)^\alpha$ for $\alpha > 0$

   (d) $f(x) = 1/(1+x^2)$

   (e) $f(x) = 1/(1-x)$

   (f) $f(x) = |x|$

   (g) $f(x) = e^{-1/(1-x^2)}$ if $|x| < 1$ and $f(x) = 0$ if $|x| \geq 1$

5. What does the Taylor series for e^x centered at $a = 0$ imply about the inequality $e^x \geq 1 + x$.

2 Problems: Improper Integrals

1.

2. Determine for which values a α does $\displaystyle\int_1^\infty \frac{1}{x\,[\ln(x)]^\alpha}\,dx$ converge.

3. Use the comparison test to determine whether the following integrals converge

   (a) $\displaystyle\int_1^\infty \frac{\sin^2(x)}{x^2}\,dx$

   (b) $\displaystyle\int_1^\infty \frac{1}{1+\sqrt{x}}\,dx$

   (c) $\displaystyle\int_0^\infty \frac{e^{-x}}{\sqrt{x}}\,dx$