

# Topics for Algebra Qualifying Exam

## Linear Algebra

Vector spaces, linear transformations and representing matrices, eigenvalues and eigenvectors, diagonalization of matrices, inner product spaces, symmetric, orthogonal, Hermitian, and unitary matrices, Jordan canonical form.

Suggested books:

- Axler, *Linear Algebra Done Right*, Chapters 1-6, 8-10
- Hoffman and Kunze, *Linear Algebra*, Chapters 1-3,7,8

## Abstract Algebra

Group Theory: Subgroups, homomorphisms, permutation groups, Sylow theorems, solvable and nilpotent groups, finitely generated abelian groups, generators and relations.

Ring Theory: Ideals, homomorphisms, fields of fractions, polynomial rings (in one and in several indeterminates), PIDs and UFDs, Chinese remainder theorem, Noetherian rings, Hilbert basis theorem, structure of semisimple Artinian rings.

Fields and Galois Theory: Normal, separable, and Galois extensions, the Fundamental Theorem of Galois Theory, Galois groups of polynomials, finite fields, solvability by radicals and applications to geometrical construction problems.

Modules: Submodules, homomorphisms and fundamental theorems of module homomorphisms, structure of finitely generated modules over PIDs, applications to linear algebra, tensor products of modules.

Suggested books:

- Dummit and Foote, *Abstract Algebra*, Chapters 1-14
- L. Grove, *Algebra*, Chapters 1-4
- Hungerford, *Algebra*, Chapters 1-5
- Jacobson's Basic Algebra Volume 1
- Lang's Algebra