# Math 519-001: Introduction to p-adic Hodge theory

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Objectives: Students will learn the fundamental ideas in p-adic Hodge theory and understand how they naturally arise in various areas of mathematics, such as number theory and algebraic geometry. In particular, students will be able to think about Galois representations from a variety of perspectives (arithmetic, geometric, or representation theoretic). Some specific topics to be covered include finite flat group schemes, p-divisible groups, Hodge-Tate decomposition for abelian varieties, Fontaine's period rings, p-adic comparison theorems, and the Fargues-Fontaine curve.

Prerequisites: abstract algebra (at the level of Math 511), with some basic knowledge on algebraic number theory and algebraic geometry recommended (ramifications of number fields, definition and basic properties of affine schemes, etc.)

Text: my notes on p-adic Hodge theory, written based on a topic course of the same title that I taught at the University of Michigan in the Winter 2020, with some supplemental texts such as Finite Flat Group Schemes and p-divisible groups by Tate, CMI summer school notes on p-adic Hodge theory by Brinon and Conrad, and Lecture notes on p-divisible groups by Demazure.

Tentative Schedule:

Lectures 1-3: Introduction - an overview of what p-adic Hodge theory is about and how it fits into a bigger picture (in relation to algebraic geometry and number theory)

Lectures 4-9: Finite flat group schemes - definitions, Cartier duality, connected-etale exact sequence, the Frobenius morphism

Lectures 10-13: p-divisible groups - definitions, Cartier duality and connected-etale exact sequence revisited, the Serre-Tate theorem on connected p-divisible groups, Dieudonne-Manin classification

Lectures 14-19: Hodge-Tate decomposition - the field Cp, the logarithm for p-divisible groups, Hodge-Tate decomposition for abelian varieties, Generic fibes of p-divisible groups

Lectures 20-22: Fontaine's formalism - definitions and properties of B-admissible representations, Hodge-Tate representations

Lectures 23-26: de-Rham representations - a brief introduction to perfectoid fields, construction of the de Rham period ring, filtered vector spaces, properties of de Rham representations, the de Rham comparison theorem (statement)

Lectures 27-30: Crystalline representations - construction of the crystalline period ring, properties of crystalline representations, the fundamental exact sequence, properties of crystalline representations

Lectures 31-36: Construction of the Fargues-Fontaine curve - untilts of a perfectoid field, two constructions of the Fargues-Fontaine curve (as a scheme and as an adic space) with a brief introduction to the theory of adic spaces

Lectures 37-43: Vector bundles on the Fargues-Fontaine curve - line bundles and their cohomology, Harder-Narasimhan formalism, stability and semistability of vector bundles, covers of the Fargues-Fontaine curve, classification of vector bundles