CERTAINTY AND CHANCE—a topics course, Spring 2021

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If I know the present state of the Universe **exactly**, can I predict its exact future? Everything that will happen, in my life, in the life of others, in politics, the fate of the Solar System, if and when the Universe will collapse on itself? Is this possible, at least in principle? Or is the future fundamentally unpredictable, depending on random events which will become known only when they occur? This old question can be asked in many different ways and in contexts, ranging from literature to physics. I do not promise a definite answer, but I offer something equally valuable: we spend a semester studying deterministic and random mathematical theories, thinking about their relation to reality. A mathematical paradigm of determinism is the theory of ordinary differential equations. We will examine this feature of the theory, focusing on its spectacular success in celestial mechanics. We will contrast it with the quantum theory, which, in its classical formulation gives randomness a central role. For this we will take a look of the fascinating mathematics of Hilbert spaces. We will also see how randomness can show up in classical physics for reasons which have nothing to do with quantum phenomena and study the related concepts of probability theory. And, while studying all this beautiful mathematics, we will keep in mind the philosophical motivation behind the question: chance or certainty? Or: does the question make sense? We will discuss it and invite some great minds—mathematical and not—to discuss it with us. I hope the course will be interesting and informative to advanced undergraduates, majoring in mathematics, physics or philosophy. Students majoring in other areas are welcome, of course.

Prerequisites: I am assuming mathematical maturity at the level of Mathematics 323—Formal Mathematical Reasoning and Writing. Students who did not take it, but are interested in the topics course—in particular, non-mathematics majors—are encouraged to discuss their background with me in advance. Prior exposure to differential equations would be helpful but is not assumed. An interest in physics is more important than formal background in this subject.
Course objectives: At the end of the course, the students will have thoroughly reviewed the theory of ordinary differential equations. The conceptual structure of the theory will be stressed, as well as selected classical applications, including planet motion, periodic phenomena and stability of oscillations. Basic concepts of probability theory will be treated in depth. In particular, the concept of a general probability space will play an important role. Simple random walk and basic theory of finite Markov chains will be studied. More advanced topics, which will be discussed without technical details or at a nontechnical level, include Hilbert spaces, entropy and its physical significance and chaotic dynamics. Students will be encouraged to read and discuss nonmathematical texts treating determinism and randomness.

Literature

Ordinary differential equations: M. Hirsch, S. Smale, R. Devaney "Differential Equations, Dynamical Systems and an Introduction to Chaos" (selected parts).
Probability theory: M. Kac "Some Stochastic Problems in Physics and Mathematics" (selected parts).
General science: D. Ruelle "Chance and Chaos".
Literature and philosophy: D. Diderot: "Jacques the Fatalist and his Master"; I. Berlin "Historical Inevitability".