The Non-commutative residue and determinants of elliptic operators
Proposal for a special topics course in the Fall of 2026

Lennie Friedlander
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The non-commutative residue of pseudodifferential operators was introduce by Manin for one-dimensional operators, and then generalized for higher dimensions by Guillemin and Wodzicki. It turned out to be a powerful tool for studying spectral asymptotics and determinants of elliptic operators. There are two approaches to the study of the non-commutative residue: an algebraic approach (Manin, Wodzicki) and a geometric approach (Guillemin). I am planning to concentrate on the geometric side of the theory, though will touch the algebraic side as well. Below are the topics that I am planning to cover, with an approximate schedule:
Weeks 1–4: a brief introduction to pseudodifferential operators: definition, products, ad-joints, change of variables, operators on manifolds.
Week 5: basics of symplectic geometry.
Weeks 6–9: non-commutative residue. Powers of elliptic operators, zeta-function of an elliptic operator and its analytic continuation.
Weeks 10–13: determinants of elliptic operators, deformation of the determinants, Polyakov formula. The determinant of the Dirac operator in dimension 2. Multiplicative anomaly Weeks 14–15: the algebraic approach.
Learning outcome: working knowledge of pseudodifferential operators, familiarity with the non-commutative residue, and the ability to apply it to spectral problems.
Prerequisites: working knowledge of manifolds, differential forms and vector fields.
I am going to write notes. For the theory of pseudodifferential operators, ”Pseudodif-ferential Operators and Spectral Theory”, by M.Shubin, is a good reference; for the rest of the material, there are no good books.