PROBABILITY IN HIGH DIMENSION—A SPECIAL TOPICS COURSE PROPOSAL

Instructor: Janek Wehr

High-dimensional probability theory studies events and probability distributions which depend on a large number of random variables, often (but not always) independent. Such objects occur naturally in numerous contexts, applied as well as theoretical. Examples include random matrices and graphs, statistics of large data sets, random codes and algorithms, random optimization problems, Markov chains and Monte Carlo methods, and, of course, machine learning. And, OF COURSE, statistical physics.

This course will be an introduction to mathematical theory underlying these applications. It will focus on a thorough treatment of concentration inequalities which can be thought of as a far-reaching generalization of the law of large numbers to a variety of nonlinear contexts. In addition to being the cornerstone of high-dimensional probability, concentration of measure is a rich and beautiful theory, related to analysis and to diffusion theory through Poincar and logarithmic Sobolev inequalities. The progress made in this field during the last 30 years allowed to solve outstanding problems in statistical mechanics and won Michel Talagrand [5] the 2024 Abel prize. The second part of the course will address suprema of stochastic processes satisfying suitable continuity assumptions. It will use combinatorial methods, subgaussian bounds and special techniques: chaining, penalization, slicing and other. Applications to gaussian processes will be presented. We will also discuss the ideas underlying Talagrand concentration of measure inequalities.

The main aim of the course is to provide the students with a working knowledge of the basics of this dynamically developing theory which will enable them to read current research literature and give them mathematical tools for applications of their choice.

The main text used in the course will be notes by Ramon van Handel [1]. Books [2] and [3] will be used as secondary references, and [4]—as a source for the requisite theory of Markov processes. [5] is the original groundbreaking work by Talagrand,

Prerequisites: a graduate probability course, such as our Math 563, would be best. As always, formal background can be substituted by genuine interest and additional reading.

Please contact me with any questions you may have.

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References

1. R. van Handel: Probability in High Dimension (2016) https://web.math.princeton.edu/ rvan/APC550.pdf

2. M. Ledoux: The Concentration of Measure Phenomenon. AMS (2001).

3. R. Vershynin: High-Dimensional Probability. Cambridge (2018).

4 D. Bakry, I. Gentil, M. Ledoux: Analysis and Geometry of Markov Diffusion Operators. Springer (2014).

5. M. Talagrand: Concentration of Measure and Isoperimetric Inequalities in Product Spaces. IHES (1995).