Pattern Formation to Hyperbolic Surfaces: An Invitation to Applied Analysis

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Applied analysis provides the rigorous mathematical architecture necessary to understand complex, real-world systems. In my work with students and collaborators, we explore how mathematical structure emerges from physical phenomena, employing tools ranging from differential geometry and abstract algebra to random matrix theory and the calculus of variations. These methods allow us to analyze diverse problems across the sciences, including self-organization in astrophysics, force chains in granular materials, and buckling instabilities in butterfly wings.

To illustrate how we bridge physical models with rigorous mathematics, the main portion of this talk will focus on a striking geometric paradox found in biology. Organisms frequently build structures with negative Gaussian curvature—such as the ruffled edges of lettuce leaves or the swimming margins of sea slugs. However, classical differential geometry presents a (seemingly) fundamental obstruction: Hilbert's Theorem strictly forbids the complete, smooth isometric immersion of the hyperbolic plane into three-dimensional Euclidean space. How, then, do these physical structures exist?

We will resolve this contradiction by exploring the delicate threshold between geometric rigidity and flexibility. By relaxing our regularity assumptions to the $C^{1,1}$ class, we can mathematically model these highly corrugated shapes. We will examine how tools from the theory of differential inclusions and convex integration, specifically focusing on branching singularities for the normal map, inform the morphologies and even the dynamics of such surfaces in nature.
This talk will assume no specialized background beyond first year classes and I hope to give an accessible introduction to open problems at the intersection of rigorous analysis and physical modeling.