

Proposal for Special Topics Courses Spring 2027 – Doug Haessig

Title: p-adic analysis and zeta functions

Description: The course will be an introduction to the study of L-functions of exponential sums and zeta functions of algebraic varieties over finite fields using p-adic analysis. The course will be broken up into the following sections:

1. Foundations of p-adic numbers and p-adic analysis
2. Introduction to L-functions and zeta functions
3. Trace formula and rationality
4. p-adic cohomology theory
5. Families of L-functions and Gauss-Manin connection
6. Dual theory (if time permits)

Texts:

- 1) Koblitz "p-adic numbers, p-adic analysis, and zeta functions"
- 2) Monsky "p-adic analysis and zeta functions"
- 3) Various papers in the literature

Prereq: Course will be kept mostly self-contained. Undergrad knowledge in algebra (some Galois but not much), linear algebra, analysis.

Learning Outcomes:

- a) Basic understanding of p-adic numbers and p-adic analysis and how it may be used to lift questions from finite fields to characteristic 0.
- b) Exposure to the Weil Conjectures on zeta functions, with a proof of the first conjecture (rationality)
- c) An introduction to Dwork's pioneering work, including his p-adic analytic lifting of an additive character over a finite field and trace formula.
- d) Introduction to p-adic cohomology theory (de Rham type) in a self-contained way.

Schedule

Weeks 1 - 3: introduction to p-adic analysis

Weeks 4 - 6: introduction to p-adic Banach space theory

Weeks 7 - 9: Dwork's trace and the rationality of the zeta function

Week 10 - 12: p-adic cohomology theory for L-functions

Week 13 - 16: Families of L-functions and Gauss-Manin connection