MATH 410

3.4.1 (a) $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ $1 > 0$
& $(2)(1) - (0)(0) = 2 > 0$
so, it is positive definite.

Inner Product $= (x_1 \; x_2) \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

$= (x_1 \quad 2x_2) \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

$= x_1 y_1 + 2 x_2 y_2$

(b) $\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$ $0 - 2 < 0$
not positive definite.

(c) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ $1 - 4 = -3 < 0$
not positive definite.

(d) $\begin{pmatrix} 5 & 3 \\ 3 & -2 \end{pmatrix}$ $-10 - 9 < 0$
not positive definite

(e) $\begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix}$ $3 - (1) = 2 > 0$
$1 > 0$ it is positive definite.

Formula Inner Product $= (x_1 \; x_2) \begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

(f) $\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$ $2 - (-1) = 3 > 0$
$1 > 0$ it is positive definite.

Formula Inner Product $= (x_1 \; x_2) \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

3.4.2 $K = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$

$q(x) = x^T K x$

$(x_1 \quad x_2) \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$(x_1 + 2x_2 \quad 2x_1 + 3x_2) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$x_1^2 + 2x_1x_2 + 2x_1x_2 + 3x_2^2$

$x_1^2 + 4x_1x_2 + 3x_2^2$

$x^+ = (1, 1)^T \qquad q(x^+) = 8 > 0$

$x^- = (-2, 1) \qquad q(x^-) = 4 - 8 + 3 = -1 < 0$

Hence, indefinite.

3.5.1 (c) $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix} = K \qquad \begin{matrix} L_{21} = 1 \\ L_{31} = 2 \end{matrix}$

$\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & -1 & -3 \end{pmatrix} \qquad L_{32} = -1$

$\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & -3 \end{pmatrix} = U \qquad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{pmatrix}$

Not +ve definite

(d) $K = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \\ 1 & -2 & 4 \end{pmatrix}$ L_{21} 1 $L_{31} = 1$

$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0 & -3 & 3 \end{pmatrix}$ $L_{32} = -3$

$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & -6 \end{pmatrix} = U$ $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -6 \end{pmatrix}$ it is not +ve definite

3.5.2 (f) $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix}$ $L_{21} = 1$ $L_{31} = 1$

$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix}$ $L_{32} = -1$ $L_{42} = 1$ $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 2 & 1 \end{pmatrix}$ $L_{43} = -2$

$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 5 \end{pmatrix} = U$ $D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix}$

Not positive definite.

4.2.4 (a) $\begin{pmatrix} 1 & b \\ b & 4 \end{pmatrix}$

if $(1)(4) - b^2 > 0$

it is positive definite

$4 - b^2 > 0$

$b^2 < 4$

$-2 > b > 2$

(b) $A = LDL^T$

$\begin{pmatrix} 1 & b \\ b & 4 \end{pmatrix}$ $L_{21} = b$

$\begin{pmatrix} 1 & b \\ 0 & 4-b^2 \end{pmatrix} = U$ $L = \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}$

$D = \begin{pmatrix} 1 & 0 \\ 0 & 4-b^2 \end{pmatrix}$ $L^T = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}$

(c) $P(x,y) = x^2 + 2bxy + 4y^2 - 2y$

$\frac{\partial P}{\partial x} = 2x + 2by = 0$ $\frac{\partial P}{\partial y} = 2bx + 8y - 2 = 0$

$$\left.\begin{aligned} 2bx + 2b^2y &= 0 \\ 2bx + 8y - 2 &= 0 \end{aligned}\right]$$

$(2b^2 - 8)y + 2 = 0 \Rightarrow y = \frac{-2}{2b^2-8} = \frac{1}{4-b^2}$

$x = \frac{-2by}{2} = -by = -b\left(\frac{1}{4-b^2}\right)$

$x = \left(\frac{b}{b^2-4}\right)$

4.4.2 (a)

Annual exp	12	14	17	21	26	30
Annual profit	60	70	90	100	100	120

$$60 = \alpha + \beta(12)$$
$$70 = \alpha + \beta(14)$$
$$\text{— — —}$$
$$\text{— ——}$$
$$120 = \alpha + \beta(30)$$

$$\begin{bmatrix} 1 & 12 \\ 1 & 14 \\ 1 & 17 \\ 1 & 21 \\ 1 & 26 \\ 1 & 30 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 60 \\ 70 \\ 90 \\ 100 \\ 100 \\ 120 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 6 & 120 \\ 120 & 2646 \end{bmatrix}$$

$$A^T \vec{b} = \begin{bmatrix} 540 \\ 11,530 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 6 & 120 & 540 \\ 120 & 2646 & 11,530 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 20 & 90 \\ 120 & 2646 & 11,530 \end{array}\right]$$

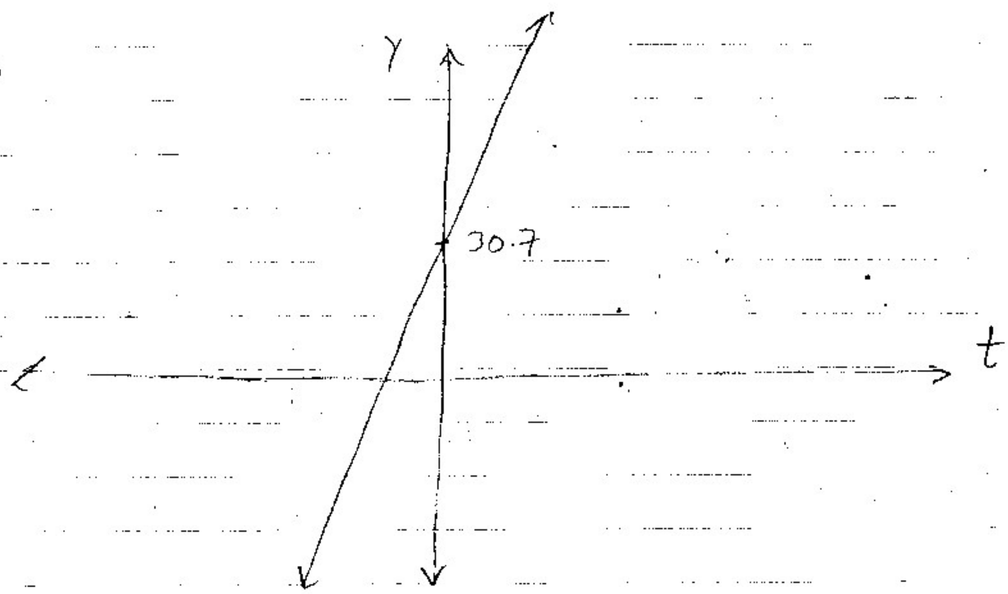
$$\left[\begin{array}{cc|c} 1 & 20 & 90 \\ 0 & 246 & 730 \end{array}\right]$$

$$\beta = \frac{730}{246} = 2.97$$

$$\alpha = 30.7$$

$$y = 30.7 + 2.97t$$

(b)



(c) $y = 30.7 + 2.97(t)$

$y = 30.7 + 2.97(50)$

$y = 179.2$

(d) $y = 30.7 + 2.97(100)$

$y = 327.7$