3.5.2 (a) $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \xrightarrow{l_{21}=2} \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{D} \underbrace{\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}}_{L^T}$

where $L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

NOT pos. def. since diagonals on D are not both > 0

In fact this matrix is INDEFINITE

(e) (I can't remember if I said c or e; they're pretty much the same!)

$\begin{pmatrix} 2 & 1 & -2 \\ 1 & 1 & -3 \\ -2 & -3 & 11 \end{pmatrix} \xrightarrow[l_{31}=-1]{l_{21}=1/2} \begin{pmatrix} 2 & 1 & -2 \\ 0 & 1/2 & -2 \\ 0 & -2 & 9 \end{pmatrix} \xrightarrow{l_{32}=-4} \underbrace{\begin{pmatrix} 2 & 1 & -2 \\ 0 & 1/2 & -2 \\ 0 & 0 & 1 \end{pmatrix}}_{U}$

$= \underbrace{\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{D} \underbrace{\begin{pmatrix} 1 & 1/2 & -1 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}}_{L^T}$ where $L = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1 & -4 & 1 \end{pmatrix}$

Matrix is POS DEF since all ~~pivots~~ diags in D > 0

3.5.5.a $x^2 + 8xy + y^2 = (x+4y)^2 - 16y^2 + y^2 = (x+4y)^2 - 15y^2$ INDEFINITE

check $K = \begin{pmatrix} 1 & 4 \\ 4 & 1 \end{pmatrix} \xrightarrow{l_{21}=4} \begin{pmatrix} 1 & 4 \\ 0 & -15 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -15 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

3.5.7 (c) $x^2+2xy+2y^2-4xz-6yz+6z^2$

$$= \vec{x}^T K \vec{x} \quad \text{where } \vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \ \& \ K = \begin{pmatrix} 1 & 1 & -2 \\ 1 & 2 & -3 \\ -2 & -3 & 6 \end{pmatrix}$$

Row reduce K: $\xrightarrow[l_{31}=-2]{l_{21}=1} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & -1 \\ 0 & -1 & 2 \end{pmatrix} \xrightarrow{l_{32}=-1} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

Diagonals are $1,1,1 > 0 \Rightarrow$ POS DEF.

4.4.1 (b) Data

t	1	2	3	4	5
y	1	0	-2	-3	-3

If $y = \alpha + \beta t$ then $\alpha+\beta(1) = 1$, $\alpha+\beta(2) = 0$, $\alpha+\beta(3) = -2$, $\alpha+\beta(4) = -3$, $\alpha+\beta(5) = -3$

I.e.
$$\underbrace{\begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} \alpha \\ \beta \end{pmatrix}}_{\vec{x}} = \underbrace{\begin{pmatrix} 1 \\ 0 \\ -2 \\ -3 \\ -3 \end{pmatrix}}_{\vec{b}}$$

NO solution! so try least squares $A^TA\vec{x} = A^T\vec{b}$

$$A^TA = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} 5 & 15 \\ 15 & 55 \end{pmatrix}$$

$$A^T \vec{b} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -2 \\ -3 \\ -3 \end{pmatrix} = \begin{pmatrix} -7 \\ -32 \end{pmatrix}$$

so $\left(\begin{array}{cc|c} 5 & 15 & -7 \\ 15 & 55 & -32 \end{array}\right) \xrightarrow{L_2 - 3L_1} \left(\begin{array}{cc|c} 5 & 15 & -7 \\ 0 & 10 & -11 \end{array}\right)$

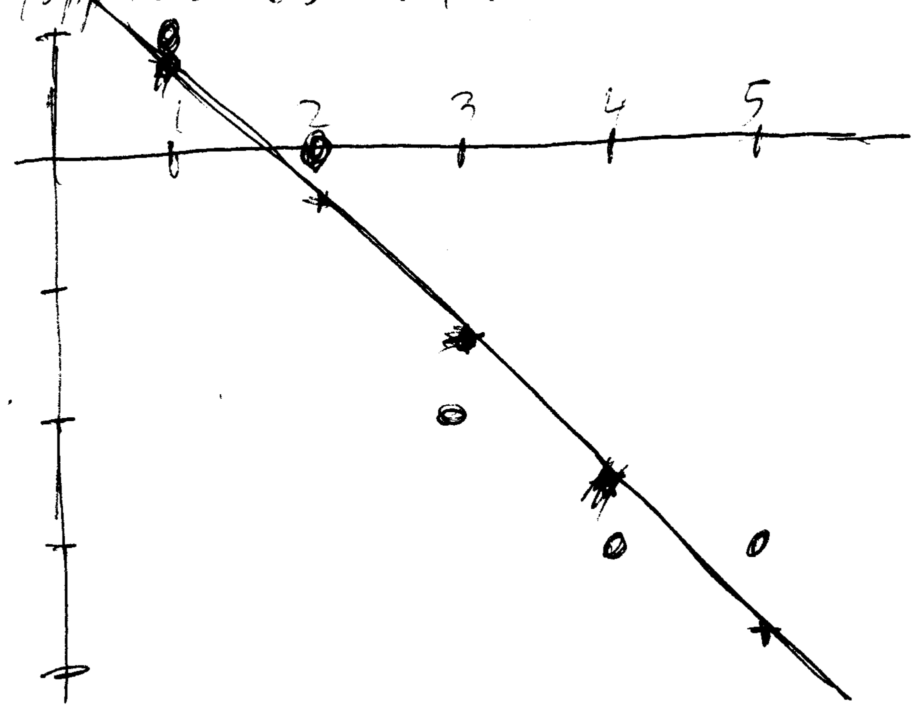
$\Rightarrow \quad 10\beta = -11 \quad \beta = -1.1$

$5\alpha + 15\beta = -7 \qquad \alpha + 3\beta = -1.4$

$\alpha = -1.4 + 3.3 = +1.9$

Check

t	1	2	3	4	5
y	1	0	-2	-3	-3
y_{app}	.8	-.3	-1.4	-2.5	-3.6



WEIGHTED LEAST SQUARES EXAMPLES

Let's redo 4.4.1 (b) if the experimenter tells us that measurements at times 2 & 4 are twice as significant as times 1, 3, 5.

We can accommodate this with a C matrix (like conductivity) and then use

$$C = \begin{pmatrix} 1 & & & & 0 \\ & 2 & & & \\ & & 1 & & \\ & & & 2 & \\ 0 & & & & 1 \end{pmatrix}$$

WEIGHTED least squares equations $A^T C A \vec{x} = A^T C \vec{b}$

Here

$$A^T C A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & & & & 0 \\ & 2 & & & \\ & & 1 & & \\ & & & 2 & \\ 0 & & & & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 4 \\ 1 & 3 \\ 2 & 8 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 21 \\ 21 & 75 \end{pmatrix}$$

and

$$A^T C \vec{b} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & & & & 0 \\ & 2 & & & \\ & & 1 & & \\ & & & 2 & \\ 0 & & & & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -2 \\ -3 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -2 \\ -6 \\ -3 \end{pmatrix} = \begin{pmatrix} -10 \\ -44 \end{pmatrix}$$

<u>SO</u>

$$\left(\begin{array}{cc|c} 7 & 21 & -10 \\ 21 & 75 & -44 \end{array}\right) \xrightarrow{l_{21}=3} \left(\begin{array}{cc|c} 7 & 21 & -10 \\ 0 & 12 & -14 \end{array}\right)$$

$\beta = -\frac{14}{12} = -\frac{7}{6}$

$\beta \approx -1.17$

$\alpha + 3\beta = -10/7 \Rightarrow \alpha = -\frac{10}{7} + \frac{7}{2} = \frac{29}{14} \approx 2.07 \approx \alpha$

Note that this α, β are close to UNWEIGHTED

$$\alpha = 1.9, \quad \beta = -1.1$$

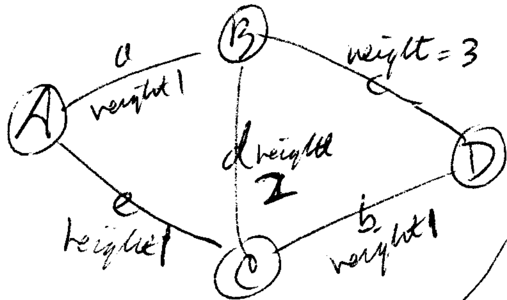
Slight difference reflects WEIGHTING of the data.

Let's redo Team Ranking problem assuming

game c is 3× as important as games a, b, e
and game d is 2× as important " " "

Then $E^TCE\vec{w} = E^TC\vec{D}$ where $C = \begin{pmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 3 & & \\ & & & 2 & \\ 0 & & & & 1 \end{pmatrix}$

Explicitly $E^TCE =$



$$\begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 6 & -2 & -3 \\ -1 & -2 & 4 & -1 \\ 0 & -3 & -1 & 4 \end{pmatrix}$$

(Recall we can look at WEIGHTS of edges that come into each node from whatever other nodes)

$$E^TC\vec{D} = \begin{pmatrix} -1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} -3 \\ 4 \\ -6 \\ -4 \\ 2 \end{pmatrix}$$

(Recall $C\vec{D}$ multiplies entry in $\vec{D}$ by entry in C)

$$= \begin{pmatrix} 1 \\ -5 \\ -6 \\ 10 \end{pmatrix}$$

So we have $\left(\begin{array}{cccc|c} 2 & -1 & -1 & 0 & 1 \\ -1 & 6 & -2 & -3 & -5 \\ -1 & -2 & 4 & -1 & -6 \\ 0 & -3 & -1 & 4 & 10 \end{array}\right)$ $L_{21} = -1/2$, $\cancel{L_{31}} = -1/2$

$$\left(\begin{array}{cccc|c} 2 & -1 & -1 & 0 & 1 \\ 0 & -\frac{11}{2} & -\frac{5}{2} & -3 & -9/2 \\ 0 & -5/2 & 7/2 & -1 & -11/2 \\ 0 & -3 & -1 & 4 & 10 \end{array}\right) \xrightarrow{L_{42}\ \ldots}$$

You get the idea!