MATH 410 HW 3 Solution

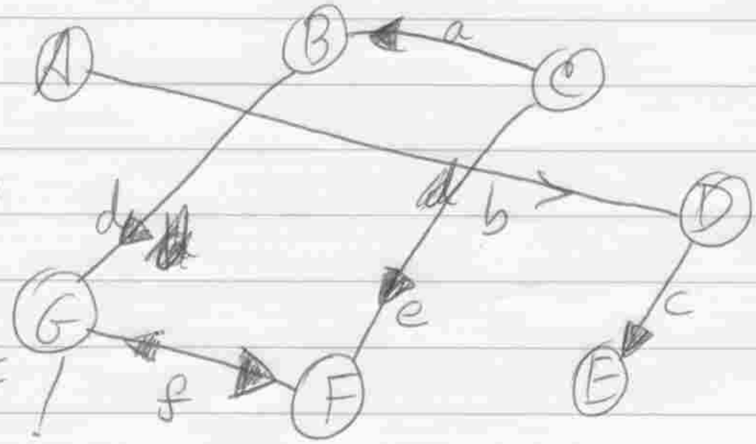
6.2.1e

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\ \begin{pmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} \end{array}$$

corresponds to the diagram

Apologies for messy drawing!

Portion A, D, E is DISCONNECTED from B, C, G, F!



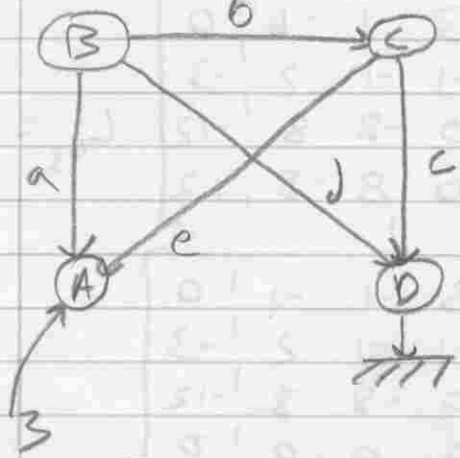
I will leave TO YOU! The process of row reducing A^T. But when you solve $A^T\vec{x}=0$ you will find

$$\vec{x} = x_6 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{pmatrix}$$

which corresponds to +a, no b, no c, +d, −e, +f which is the loop

$C \to B \to G \to F$

6.2.2.



$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \end{array}$$

Total Into A $\rightarrow 3 + I_a + I_e = 0$

$I_d + I_c - 3 = 0$

$$-A^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \qquad -A^T A \vec{V} = \begin{bmatrix} -3 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$

$$-A^T A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

$$-\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 0 \\ 3 \end{bmatrix} \Rightarrow \left[\begin{array}{cccc|c} 2 & -1 & -1 & 0 & 3 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 3 & -1 & 0 \\ 0 & -1 & -1 & 2 & -3 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{cccc|c} -1 & 3 & -1 & -1 & 0 \\ 0 & 5 & -3 & -2 & 3 \\ 0 & -4 & 4 & 0 & 0 \\ 0 & -1 & -1 & 2 & -3 \end{array}\right] \leftarrow \left[\begin{array}{cccc|c} -1 & 3 & -1 & -1 & 0 \\ 2 & -1 & -1 & 0 & 3 \\ -1 & -1 & 3 & -1 & 0 \\ 0 & -1 & -1 & 2 & -3 \end{array}\right]$$

$L_{21} = -2 \quad L_{31} = -1$

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$$\left[\begin{array}{cccc|c} -1 & 3 & -1 & -1 & 0 \\ 0 & -1 & -1 & 2 & -3 \\ 0 & 5 & -3 & -2 & 3 \\ 0 & -4 & 4 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} -1 & 3 & -1 & -1 & 0 \\ 0 & -1 & -1 & 2 & -3 \\ 0 & 0 & -8 & 8 & -12 \\ 0 & 0 & 8 & -8 & 12 \end{array}\right] \quad L_{43} = -1$$

$L_{32} = -5 \quad L_{42} = 4$

$$\downarrow \left[\begin{array}{cccc|c} -1 & 3 & -1 & -1 & 0 \\ 0 & -1 & -1 & 2 & -3 \\ 0 & 0 & -8 & 8 & -12 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$8V_D - 8V_C = -12 \qquad 2V_D - V_C - V_B = -3$

$-V_D - V_C + 3V_B - V_A = 0$

$\boxed{V_C = V_D + 1.5}$ back subst. $2V_D - V_D - 1.5 - V_B = -3$

$\boxed{V_B = V_D + 1.5}$

$-V_D - V_D - 1.5 + 3V_D + 4.5 - V_A = 0$

$V_A = V_D + 3 \qquad V = \begin{bmatrix} V_D + 3 \\ V_D + 1.5 \\ V_D + 1.5 \\ V_D \end{bmatrix} = \begin{bmatrix} 3 \\ 1.5 \\ 1.5 \\ 0 \end{bmatrix} + V_D \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

☆ Prof. B did min length for 1.8.2ⓐ in class. Look at your notes!

Minimum length 1.8.3(d)

$$\vec{x} = \begin{bmatrix} 3x_2 - 2x_3 + 1 \\ x_2 \\ x_3 \end{bmatrix} = \text{length}^2 =$$

$$f(x,y) = 9x_2^2 + 4x_3^2 + 1 + 6x_2 - 4x_3 - 12x_2x_3 + x_2^2 + x_3^2$$

$$f(x_2, x_3) = 10x_2^2 + 5x_3^2 - 12x_2x_3 - 4x_3 + 6x_2 + 1$$

$$\frac{\partial f}{\partial x_2} = 20x_2 - 12x_3 + 6 = 0$$

$$\frac{\partial f}{\partial x_3} = 10x_3 - 12x_2 - 4 = 0$$

$$\left[\begin{array}{cc|c} 20 & -12 & -6 \\ -12 & 10 & 4 \end{array}\right] \rightarrow \text{Divide Both rows by 2}$$

$$\left[\begin{array}{cc|c} 10 & -6 & -3 \\ 0 & 1.4 & 0.2 \end{array}\right] \xleftarrow{l_{21} = -0.6} \left[\begin{array}{cc|c} 10 & -6 & -3 \\ -6 & 5 & 2 \end{array}\right]$$

$$1.4x_3 = 0.2$$

$$x_3 = \frac{1}{7} \rightarrow \text{back substitute} \rightarrow 10x_2 - 6\left(\frac{1}{7}\right) = -3$$

$$10x_2 - \frac{6}{7} = -3$$

$$10x_2 = \frac{-15}{7}$$

$$x_2 = \frac{-3}{14}$$

$$\vec{X}_{M1} = \begin{bmatrix} -\frac{9}{14} - \frac{2}{7} + 1 \\ -\frac{3}{14} \\ \frac{1}{7} \end{bmatrix} = \begin{bmatrix} \frac{1}{14} \\ -\frac{3}{14} \\ \frac{1}{7} \end{bmatrix}$$

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Using $(AA^T)u = b$

$$A = \begin{bmatrix} 2 & -6 & 4 \\ -1 & 3 & -2 \end{bmatrix}_{2\times 3} \quad A^T = \begin{bmatrix} 2 & -1 \\ -6 & 3 \\ 4 & -2 \end{bmatrix}_{3\times 2}$$

$$AA^T = \begin{bmatrix} 56 & -28 \\ -28 & 14 \end{bmatrix}_{2\times 2}$$

$(AA^T)u = b$

$$\begin{bmatrix} 28 & -14 & | & 1 \\ -28 & 14 & | & -1 \end{bmatrix} \leftarrow \begin{bmatrix} 56 & -28 & | & 2 \\ -28 & 14 & | & -1 \end{bmatrix}$$

$\downarrow L_{21} = -1$

$$\begin{bmatrix} 28 & -14 & | & 1 \\ 0 & 0 & | & 0 \end{bmatrix}$$

$$28u_1 - 14u_2 = 1$$
$$28u_1 = 14u_2 + 1$$
$$u_1 = \frac{1}{2}u_2 + \frac{1}{28}$$

u_2 free $\Rightarrow \infty$ # of u solns

$$x_{MI} = A^T u = \begin{bmatrix} 2 & -1 \\ -6 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{2}u_2 + \frac{1}{28} \\ u_2 \end{bmatrix} = \begin{bmatrix} u_2 + \frac{1}{14} - u_2 \\ -3u_2 - \frac{3}{14} + 3u_2 \\ 2u_2 + \frac{1}{7} - 2u_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{14} \\ -\frac{3}{14} \\ \frac{1}{7} \end{bmatrix}$$