HW4 SOLUTIONS

3.5.2(c)

$$\begin{bmatrix} 3 & -1 & 3 \\ -1 & 5 & 1 \\ 3 & 1 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 5 & 1 \\ 3 & -1 & 3 \\ 3 & 1 & 5 \end{bmatrix} \xrightarrow[L_{31}=-3]{L_{21}=-3} \begin{bmatrix} -1 & 5 & 1 \\ 0 & 14 & 6 \\ 0 & 14 & 8 \end{bmatrix} \xrightarrow{L_{32}=1} \begin{bmatrix} -1 & 5 & 1 \\ 0 & 14 & 6 \\ 0 & 0 & 2 \end{bmatrix}$$

$$D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix} \qquad L^T = \begin{bmatrix} 1 & -3 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$LDL^T = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -3 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & 3 \\ 0 & 14 & 14 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 3 & 3 \\ 3 & 5 & 5 \\ 3 & 5 & 7 \end{bmatrix}$$

since diagonals of D not all pos. or neg. This matrix is indefinite

3.5.7 (c)

$$x^2+2xy+2y^2-4xz-6yz+6z^2=$$

$$\begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 1 & 1 & -2 \\ 1 & 2 & -3 \\ -2 & -3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Row Reduce

$$\begin{array}{l} \\ L_{21}=1 \\ L_{31}=-2 \end{array}\begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & -5 \\ 0 & -1 & 2 \end{bmatrix} \xrightarrow[L_{32}=-1]{} \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & -5 \\ 0 & 0 & -3 \end{bmatrix}$$

indefinite

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

4.4.1(c)

t_i	-2	-1	0	1	2
y_i	-5	-3	-2	0	3

$y = \alpha + \beta t$

$$-5 = \alpha + \beta\cdot(-2)$$
$$-3 = \alpha + \beta(-1)$$
$$-2 = \alpha + \beta(0)$$
$$0 = \alpha + \beta(1)$$
$$3 = \alpha + \beta(2)$$

$$\underbrace{\begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \alpha \\ \beta \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -5 \\ -3 \\ -2 \\ 0 \\ 3 \end{bmatrix}}_{b}$$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 10 \end{bmatrix}$$

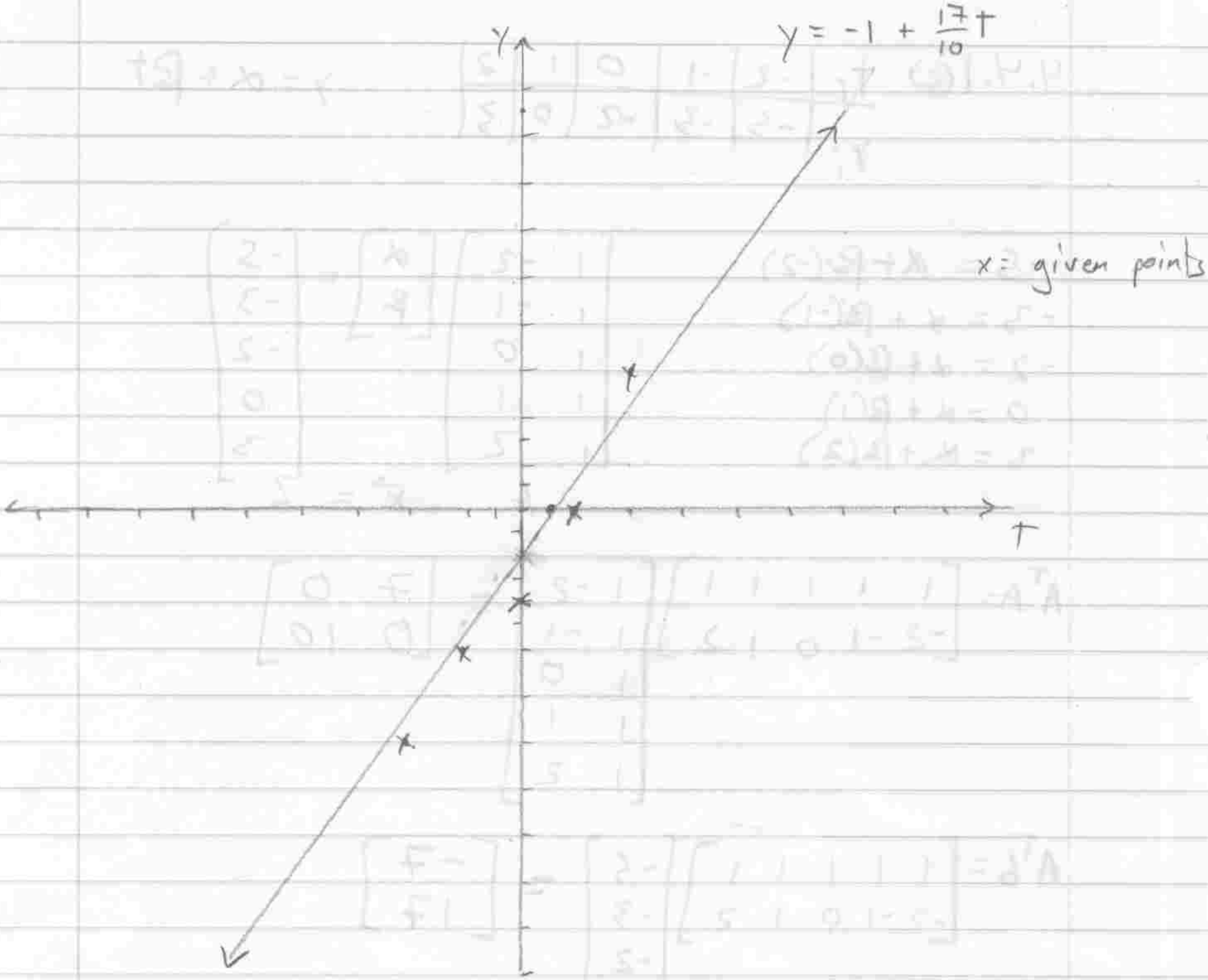
$$A^T b = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} -5 \\ -3 \\ -2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} -7 \\ 17 \end{bmatrix}$$

$\vec{x}_{ml}$ = soln to

$$\left[\begin{array}{cc|c} 7 & 0 & -7 \\ 0 & 10 & 17 \end{array}\right] \Rightarrow \begin{array}{l} \alpha = -1 \\ \beta = \frac{17}{10} \end{array}$$

$$y = -1 + \frac{17}{10} t$$

4



4.4.2

Annual ad expenditure	12	14	17	21	26	30
Annual profit	60	70	90	100	100	120

(a) $60 = \alpha + \beta(12)$
$70 = \alpha + \beta(14)$
$90 = \alpha + \beta(17)$
$100 = \alpha + \beta(21)$
$100 = \alpha + \beta(26)$
$120 = \alpha + \beta(30)$

$$\begin{bmatrix} 1 & 12 \\ 1 & 14 \\ 1 & 17 \\ 1 & 21 \\ 1 & 26 \\ 1 & 30 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 60 \\ 70 \\ 90 \\ 100 \\ 100 \\ 120 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 12 & 14 & 17 & 21 & 26 & 30 \end{bmatrix} \begin{bmatrix} 1 & 12 \\ 1 & 14 \\ 1 & 17 \\ 1 & 21 \\ 1 & 26 \\ 1 & 30 \end{bmatrix} = \begin{bmatrix} 6 & 120 \\ 120 & 2646 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 12 & 14 & 17 & 21 & 26 & 30 \end{bmatrix} \begin{bmatrix} 60 \\ 70 \\ 90 \\ 100 \\ 100 \\ 120 \end{bmatrix} = \begin{bmatrix} 540 \\ 11,530 \end{bmatrix}$$

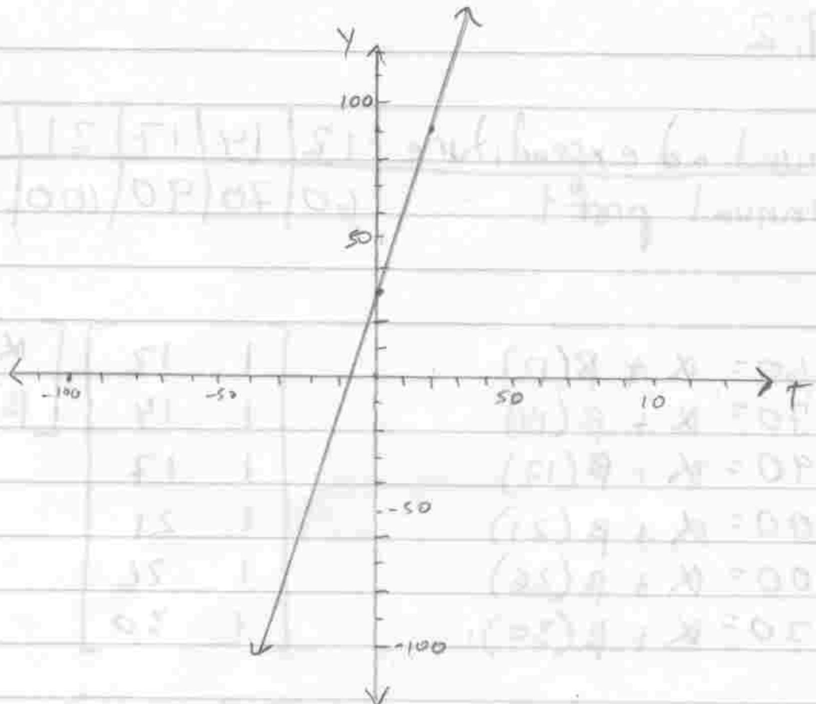
$$\vec{x}_{"1"} = \left[\begin{array}{cc|c} 6 & 120 & 540 \\ 120 & 2646 & 11,530 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 20 & 90 \\ 120 & 2646 & 11,530 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 20 & 90 \\ 0 & 246 & 730 \end{array}\right]$$

$\alpha = 30.7$ (th.)
$\beta = 2.97$ (th.)

$$\boxed{y = 30.7 + 2.97t}$$

(b)



(c) $y = 30.7 + 2.97(20)$

$y = 90.1$

(d) $y = 30.7 + 2.97(50)$

$y = 179.2$

Team Ranking

$$\begin{array}{c} \\ a \\ b \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \end{array}\begin{pmatrix} P_A \\ P_B \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$ NO SOLUTION!

$$A^TA = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix} \qquad A^T\vec{b} = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

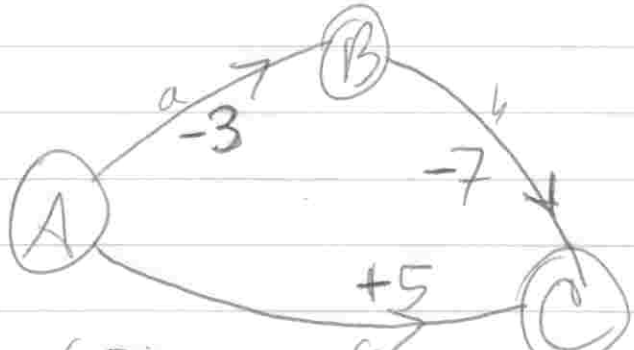
$$\left(\begin{array}{cc|c} 2 & -2 & -2 \\ -2 & 2 & 2 \end{array}\right) \to \left(\begin{array}{cc|c} 2 & -2 & -2 \\ 0 & 0 & 0 \end{array}\right)$$

$\Rightarrow$ P_B free $\quad 2P_A - 2P_B = -2 \quad$ $P_A = P_B - 1$

$$\Rightarrow \vec{p} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} + P_B\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$ A lower than B

NOTE: Ranking of teams depends ~~mostly~~ only on particular solution $\vec{p}^{(p)}$, NOT complementary part!

Real Ranking: (a)



Let $P_A = x$
$P_B = y$
$P_C = z$

$$\begin{array}{c} a \\ b \\ c \end{array}\begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ -7 \\ +5 \end{pmatrix}$$

NO SOLUTION!

Least squares $A^TA = \begin{pmatrix}-1&0&-1\\1&-1&0\\0&1&1\end{pmatrix}\begin{pmatrix}-1&1&0\\0&-1&1\\-1&0&1\end{pmatrix} = \begin{pmatrix}2&-1&-1\\-1&2&-1\\-1&-1&2\end{pmatrix}$

$$A^T\vec{b} = \begin{pmatrix}-1&0&-1\\1&-1&0\\0&1&1\end{pmatrix}\begin{pmatrix}-3\\-7\\5\end{pmatrix} = \begin{pmatrix}-2\\4\\-2\end{pmatrix}$$

A.N. $\left(\begin{array}{ccc|c}2&-1&-1&-2\\-1&2&-1&4\\-1&-1&2&-2\end{array}\right) \xrightarrow[\ell_{31}=-1/2]{\ell_{21}=-1/2} \left(\begin{array}{ccc|c}2&-1&-1&-2\\0&3/2&-3/2&3\\0&-3/2&3/2&-3\end{array}\right)$

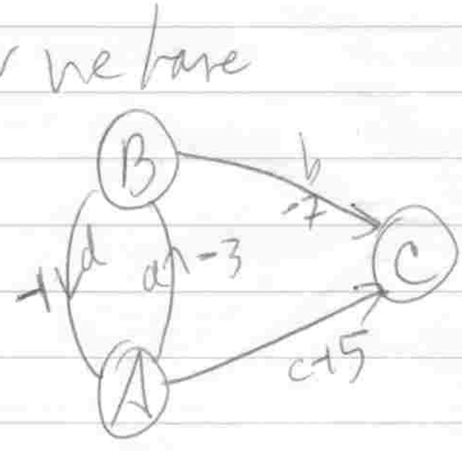
$\rightarrow \left(\begin{array}{ccc|c}2&-1&-1&-2\\0&3/2&-3/2&3\\0&0&0&0\end{array}\right)$ z free, $\frac{3}{2}y - \frac{3}{2}z = 3$

OK! $\rightarrow$ $y = 2 + z$

$2x - y - z = -2 \quad \Rightarrow 2x = -\cancel{2}+\cancel{2}+2z \quad x = z$

so $\vec{x} = \begin{pmatrix}0\\2\\0\end{pmatrix} + z\begin{pmatrix}1\\1\\1\end{pmatrix}$ B leads, A & C tied!

(b) Now we have



$$\begin{array}{c}a\\b\\c\\d\end{array}\left(\begin{array}{ccc|c}-1&1&0&-3\\0&-1&1&-7\\-1&0&1&+5\\1&-1&0&1\end{array}\right)$$

Then $A^TA = \begin{pmatrix} -1 & 0 & -1 & 1 \\ 1 & -1 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & -2 & -1 \\ -2 & 3 & -1 \\ -1 & -1 & 2 \end{pmatrix}$

$A^T\vec{b} = \begin{pmatrix} -1 & 0 & -1 & 1 \\ 1 & -1 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ -7 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$

A.M. $\left(\begin{array}{ccc|c} 3 & -2 & -1 & -1 \\ -2 & 3 & -1 & 3 \\ -1 & -1 & 2 & -2 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} -1 & -1 & 2 & -2 \\ -2 & 3 & -1 & 3 \\ 3 & -2 & -1 & -1 \end{array}\right) \xrightarrow[\ell_{31}=-3]{\ell_{21}=2}$

$\left(\begin{array}{ccc|c} -1 & -1 & 2 & -2 \\ 0 & 5 & -5 & 7 \\ 0 & -5 & 5 & -7 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} -1 & -1 & 2 & -2 \\ 0 & 5 & -5 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right) \leftarrow$ ok!

z free

$5y - 5z = 7 \qquad y = z + 7/5$

$-x - y + 2z = -2 \qquad x = 2 - (z + 7/5) + 2z$

$= 3/5 + z$

$\vec{x} = \begin{pmatrix} 3/5 \\ 7/5 \\ 0 \end{pmatrix} + z \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\boxed{\text{B Leads, A next, C last}}$

$\boxed{\text{Q: HOW COME A losing to B INCREASES A's rank? THINK ABOUT IT!}}$