



The problem is to find the $\left(\frac{\text{Tension}}{\text{Length}}\right)'s = \tau$ in each beam a, b, and c.

They satisfy

$$\begin{pmatrix} -2 \\ -1 \end{pmatrix} \tau_a + \begin{pmatrix} -3 \\ 0 \end{pmatrix} \tau_c = \begin{pmatrix} 0 \\ F \end{pmatrix} \quad \text{Equil. at NODE ①}$$

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \tau_a + \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tau_b = \begin{pmatrix} 0 \\ -600 \end{pmatrix} \quad \text{Equil at NODE ②}$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} \tau_b + \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tau_c = \begin{pmatrix} 0 \\ G \end{pmatrix} \quad \text{Equil at node ③}$$

OR

$$\begin{pmatrix} -2 & 0 & -3 \\ -1 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 3 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} \tau_a \\ \tau_b \\ \tau_c \end{pmatrix} = \begin{pmatrix} 0 \\ F \\ 0 \\ -600 \\ 0 \\ G \end{pmatrix}$$

These equations have a lot of zeros, so it would be hard to solve them,

ESPECIALLY if we do some rearranging!

ORIGINAL AUGMENTED MATRIX IS

(2)

$$\left(\begin{array}{ccc|c} -2 & 0 & -3 & 0 \\ -1 & 0 & 0 & F \\ 2 & -1 & 0 & 0 \\ 1 & 1 & 0 & -600 \\ 0 & 1 & 3 & 0 \\ 0 & -1 & 0 & G \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} -1 & 0 & 0 & F \\ 0 & -1 & 0 & G \\ 0 & 0 & -3 & -2F \\ 0 & 0 & 0 & 2F - G \\ 0 & 0 & 0 & F + G - 600 \\ 0 & 0 & 0 & G - 2F \end{array} \right)$$

is clearly
going to be
way simpler
now:

$$\rightarrow \left(\begin{array}{ccc|c} -1 & 0 & 0 & F \\ 0 & -1 & 0 & G \\ 0 & 0 & -3 & -2F \\ 0 & 0 & 0 & 2F - G \\ 0 & 0 & 0 & F + G - 600 \\ 0 & 0 & 0 & G - 2F \end{array} \right)$$

where I did several
steps in the same
matrix, but did not
swap any more rows.

THEFORE a solution exists provided $G = 2F$, $F + G = 600$

$$\Rightarrow 3F = 600 \quad F = 200 \quad G = 400$$

(This is AN OVERDETERMINED linear system!)

From the first 3 rows we have $-\tau_a = F \Rightarrow \tau_a = -200$
(the - indicates COMPRESSION instead of TENSION)

and $-\tau_b = 400 \Rightarrow \tau_b = -400$ (also COMPRESSION)

AND finally $-3\tau_c = -400 \Rightarrow \tau_c = 400/3$

(*) To get the actual tensions, multiply the σ 's by the beam lengths (3)

$$L_a = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$L_b = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$L_c = \sqrt{3^2 + 0^2} = 3$$

$$\Rightarrow T_a = L_a \sigma_a = -200\sqrt{5} \approx -447$$

$$T_b = L_b \sigma_b = -400\sqrt{2} \approx -565.7 = -566$$

$$T_c = L_c \sigma_c = 3(400/3) = +400$$

These would be in units of POUNDS if $W = 600$ pounds.

(*) This was NOT NECESSARY for the homework!

But it is good to see how to translate results from abstract numbers into physical quantities.