George FRANCIS III

HW #7

$\begin{pmatrix} 8.5.1e \\ 8.5.2e \end{pmatrix}$ $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix}$ $A^T = \begin{bmatrix} 2 & 0 \\ 1 & -2 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$ $\sigma_1 = \sqrt{7}$ $\sigma_2 = \sqrt{3}$ $\Sigma = \begin{pmatrix} \sqrt{7} & 0 \\ 0 & \sqrt{3} \end{pmatrix}$

$AA^T = \begin{bmatrix} 5 & -2 \\ -2 & 5 \end{bmatrix}$ $\quad P_A: (5-\lambda)(5-\lambda) - 4 = \lambda^2 - 10\lambda + 21 = (\lambda-7)(\lambda-3) \Rightarrow \begin{matrix} \lambda_1 = 7 \\ \lambda_2 = 3 \end{matrix}$

for $\lambda_2 = 3$ $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \to \begin{bmatrix} 2 & -2 \\ 0 & 0 \end{bmatrix}$ $x = y$ $\vec{x} = y\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\vec{\xi}^{\,2}_{AA^T} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

For $\lambda_1 = 7$ $\begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix} \to \begin{bmatrix} -2 & -2 \\ 0 & 0 \end{bmatrix}$ $x = -y$ $\vec{x} = y\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $\vec{\xi}^{\,1}_{AA^T} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\vec{q}^{\,2}_{AA^T} = \dfrac{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}{\sqrt{2}}$ $\qquad Q_{AA^T} = \dfrac{\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}}{\sqrt{2}} = Q^T_{AA^T}$ $\qquad \bigstar \quad \Sigma = \begin{pmatrix} \sqrt{7} & 0 \\ 0 & \sqrt{3} \end{pmatrix}$

$\vec{q}^{\,1}_{AA^T} = \dfrac{\begin{pmatrix} -1 \\ 1 \end{pmatrix}}{\sqrt{2}}$ $\qquad \Sigma^{-1} = \begin{pmatrix} \frac{\sqrt{7}}{7} & 0 \\ 0 & \frac{\sqrt{3}}{3} \end{pmatrix}$

$A^TA = \begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 5 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix}$ $\quad P_A = \lambda^2(\lambda-7)(\lambda-3)$ $\quad \lambda_1 = 7$, $\lambda_2 = 3$, $\lambda_3 = 0$ zeros can be ignored $\bigstar$

For $\underset{4\times1}{\vec{\xi}^{\,1}_{A^TA}} = \underset{4\times2}{A^T}\underset{2\times1}{\vec{\xi}^{\,1}_{AA^T}} = \begin{bmatrix} 2 & 0 \\ 1 & -2 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ 0 \\ 1 \end{bmatrix} = \vec{\xi}^{\,1}_{A^TA}$ $\quad \vec{q}^{\,1}_{A^TA} = \frac{1}{\sqrt{14}}\begin{bmatrix} -2 \\ -3 \\ 0 \\ 1 \end{bmatrix}$ $\quad Q_{A^TA} = \begin{bmatrix} \frac{-2}{\sqrt{14}} & \frac{2}{\sqrt{6}} \\ \frac{-3}{\sqrt{14}} & -\frac{1}{\sqrt{6}} \\ 0 & 0 \\ \frac{1}{\sqrt{14}} & \frac{1}{\sqrt{6}} \end{bmatrix}$

$\vec{\xi}^{\,2}_{A^TA} = A^T\vec{\xi}^{\,2}_{AA^T} = \begin{bmatrix} 2 & 0 \\ 1 & -2 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix} = \vec{\xi}^{\,2}_{A^TA}$ $\quad \vec{q}^{\,2}_{A^TA} = \frac{1}{\sqrt{6}}\begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ $\quad Q^T_{A^TA} = \begin{bmatrix} \frac{-2}{\sqrt{14}} & \frac{-3}{\sqrt{14}} & 0 & \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{6}} \end{bmatrix}$

SVD of A and A^T

$A = Q_{AA^T}\begin{pmatrix} \sqrt{7} & 0 \\ 0 & \sqrt{3} \end{pmatrix}Q^T_{A^TA} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix}$

$A^T = Q_{A^TA}\begin{pmatrix} \sqrt{7} & 0 \\ 0 & \sqrt{3} \end{pmatrix}Q^T_{AA^T} = \begin{bmatrix} 2 & 0 \\ 1 & -2 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$ $\qquad A^+ = Q_{A^TA}\,\Sigma^{-1}\,Q^T_{AA^T} = \begin{bmatrix} 10/21 & 4/21 \\ 1/21 & -8/21 \\ 0 & 0 \\ 2/21 & 5/21 \end{bmatrix}$

EXTRA PROBLEM

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} = A^T \qquad AA^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix} = A^TA \qquad P_A = -\lambda(\lambda-4)(\lambda-1)$$

$\lambda_1 = 4$
$\lambda_2 = 1$
$\lambda_3 = 0$

using

$\lambda_1 = 4$

$$\begin{bmatrix} -3 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -3 \end{bmatrix} \xrightarrow{L_{21} = -\frac{1}{3},\ L_{31} = -\frac{1}{3}} \begin{bmatrix} -3 & 1 & 1 \\ 0 & -2/3 & 4/3 \\ 0 & 4/3 & -8/3 \end{bmatrix} \to \begin{bmatrix} -3 & 1 & 1 \\ 0 & -2/3 & 4/3 \\ 0 & 0 & 0 \end{bmatrix}$$

$x = \frac{2z + z}{3} = z$
$y = \frac{3}{2}\left(\frac{4}{3}\right)z = 2z$
$z = z$

$$\vec{x} = z\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$\lambda_2 = 1$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \xrightarrow{\text{swap},\ L_{21}=1} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x = -2y - z = -3z$
$y = z$
$z = z$

$$\vec{x} = \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} \sqrt{4} & 0 \\ 0 & \sqrt{1} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\vec{q}^{\,1}_{A^T} = \frac{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}}{\sqrt{6}} = \begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix} = \vec{q}^{\,1}_{AA^T}$$

$$Q_{AA^T} = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{-3}{\sqrt{11}} \\ \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{11}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{11}} \end{pmatrix} = Q_{A^TA}$$

$$\vec{q}^{\,2}_{A^T} = \frac{\begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}}{\sqrt{11}} = \begin{pmatrix} \frac{-3}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} \end{pmatrix} = \vec{q}^{\,2}_{AA^T}$$

$$Q^T_{A^TA} = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{-3}{\sqrt{11}} & \frac{1}{\sqrt{11}} & \frac{1}{\sqrt{11}} \end{pmatrix}$$

need $A^{exact} = Q_{AA^T} \Sigma Q^T_{A^TA}$ removing higher values

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{\sqrt{6}} & \cancel{\frac{-3}{\sqrt{11}}} \\ \frac{2}{\sqrt{6}} & \cancel{\frac{1}{\sqrt{11}}} \\ \frac{1}{\sqrt{6}} & \cancel{\frac{1}{\sqrt{11}}} \end{pmatrix}}_{3\times 1} \underbrace{\begin{pmatrix} 2 & \cancel{0} \\ \cancel{0} & \cancel{1} \end{pmatrix}}_{1\times 1} \underbrace{\begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \cancel{\frac{-3}{\sqrt{11}}} & \cancel{\frac{1}{\sqrt{11}}} & \cancel{\frac{1}{\sqrt{11}}} \end{pmatrix}}_{1\times 3} = \begin{pmatrix} 1/3 & 2/3 & 1/3 \\ 2/3 & 4/3 & 2/3 \\ 1/3 & 2/3 & 1/3 \end{pmatrix}$$

10.4.1 g $M = \begin{bmatrix} .1 & .5 & 0 \\ .1 & .2 & 1 \\ .8 & .3 & 0 \end{bmatrix}$ $\quad P_A = -(\lambda-1)(\lambda^2 + .7 + .37)$ $\quad \dfrac{-.7 \pm \sqrt{.7^2 - 4(.37)}}{2} =$

$\lambda_2, \lambda_3 \simeq -3.5 \pm \dfrac{i}{2}$

$\lambda_1 = 1$

$$M - I = \begin{bmatrix} .1-1 & .5 & 0 \\ .1 & .2-1 & 1 \\ .8 & .3 & -1 \end{bmatrix} = \begin{bmatrix} -.9 & .5 & 0 \\ .1 & -.8 & 1 \\ .8 & .3 & -1 \end{bmatrix} \begin{matrix} L_{21} = \frac{1}{-9} \\ = \\ L_{31} = -1.125 \end{matrix} \begin{bmatrix} -9/10 & 1/2 & 0 \\ 0 & -67/90 & 1 \\ 0 & 67/90 & -1 \end{bmatrix}$$

$$\to \begin{bmatrix} -9/10 & 1/2 & 0 \\ 0 & -67/90 & 1 \\ 0 & 0 & 0 \end{bmatrix} \to$$

$z = \text{free}$ $\quad x = \frac{10}{18} y = \frac{5}{9}\left(\frac{90}{67} z\right) = \frac{50}{67} z$

$y = \frac{90}{67} z$

$z = z$

$$\vec{x} = z \begin{pmatrix} 50/67 \\ 90/67 \\ 1 \end{pmatrix}$$

to interpert as probabilities, need:

$$\text{Sum} = \left(\frac{50}{67} + \frac{90}{67} + 1\right) = \left(\frac{207}{67}\right) z = 1 \qquad \boxed{z = \frac{67}{207}} \simeq .324$$

$$\vec{x} = \left(\frac{67}{207}\right) \begin{pmatrix} 50/67 \\ 90/67 \\ 1 \end{pmatrix} = \begin{bmatrix} 50/207 \\ 10/23 \\ 67/207 \end{bmatrix}$$

— OR —

$$\text{Sum} = \left[50/207 + 10/23 + 67/207 \right] = 1$$

$$u^* = \frac{\begin{pmatrix} 50/67 \\ 90/67 \\ 1 \end{pmatrix}}{\left(\frac{207}{67}\right)} = \begin{bmatrix} 50/207 \\ 10/23 \\ 67/207 \end{bmatrix}$$

10.4.1 m

$$T = \begin{bmatrix} .1 & .3 & .7 & 0 \\ .1 & .2 & 0 & .8 \\ 0 & .5 & 0 & .2 \\ .8 & 0 & .3 & 0 \end{bmatrix} \qquad T - I = \begin{bmatrix} -.9 & .3 & .7 & 0 \\ .1 & -.8 & 0 & .8 \\ 0 & .5 & -1 & .2 \\ .8 & 0 & .3 & -1 \end{bmatrix} =$$

$L_{21} = -\frac{1}{9}$

$L_{41} = -\frac{8}{9}$

$$\begin{bmatrix} -9/10 & 3/10 & 7/10 & 0 \\ 0 & -23/30 & 7/90 & 4/5 \\ 0 & 5/10 & -1 & 2/10 \\ 0 & 4/15 & 83/90 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} -9/10 & 3/10 & 7/10 & 0 \\ 0 & -23/30 & 7/90 & 4/5 \\ 0 & 0 & -\frac{131}{138} & \frac{83}{115} \\ 0 & 0 & \frac{131}{138} & -\frac{83}{115} \end{bmatrix} \rightarrow \begin{array}{cccc} w & x & y & z \end{array} \begin{bmatrix} -9/10 & 3/10 & 7/10 & 0 \\ 0 & -23/30 & 7/90 & 4/5 \\ 0 & 0 & -\frac{131}{138} & \frac{83}{115} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$L_{32} = -\frac{30}{46}$

$L_{42} = -\frac{120}{345} = -\frac{8}{23}$

z Free

$$y = \left(\frac{138}{131}\right)\frac{83}{115} z = \frac{498}{655} z$$

$$x = \frac{\frac{7}{90} y + \frac{4}{5} z}{\frac{23}{30}} = \frac{\frac{7}{90}\left(\frac{498}{655}\right) z + \frac{4}{5} z}{\left(\frac{23}{30}\right)} = \frac{734}{655} z$$

$$w = \frac{\left(3/10\, x + 7/10\, y\right)}{9/10} = \frac{\left(3/10\right)\left(\frac{734}{655}\right) z + \left(7/10\right)\left(\frac{498}{655}\right) z}{9/10} = \left(\frac{632}{655}\right) z$$

$$\vec{x} = z \begin{pmatrix} \frac{632}{655} \\ \frac{734}{655} \\ \frac{498}{655} \\ 1 \end{pmatrix} \qquad \text{Sum} = \frac{632 + 734 + 498 + 655}{655} = \frac{2519}{655} z = 1$$

$$z = \frac{655}{2519}$$

$$u^* = \begin{bmatrix} \frac{632}{2519} \\ \frac{734}{2519} \\ \frac{498}{2519} \\ \frac{655}{2519} \end{bmatrix} \qquad \text{Sum} = 1$$

10.4.2

$$M = \begin{array}{cc} & \overbrace{\begin{array}{ccc} F & BC & WC \end{array}}^{\text{Father}} \\ \text{Son}\left\{\begin{array}{c} F \\ BC \\ WC \end{array}\right. & \begin{bmatrix} .4 & .1 & .05 \\ .3 & .3 & .25 \\ .3 & .6 & .7 \end{bmatrix} \end{array}$$

$$M - I = \begin{bmatrix} -.6 & .1 & .05 \\ .3 & -.7 & .25 \\ .3 & .6 & -.3 \end{bmatrix} \xrightarrow[l_{31} = -\frac{1}{2}]{l_{21} = -\frac{1}{2}} \begin{bmatrix} -.6 & .1 & .05 \\ 0 & -.65 & .275 \\ 0 & .65 & -.275 \end{bmatrix}$$

$$\begin{bmatrix} -.6 & .1 & .05 \\ 0 & -.65 & .275 \\ 0 & 0 & 0 \end{bmatrix} \quad z \text{ FREE}$$

$$x = \frac{.1y + .05z}{.6} = \frac{.1\left(\frac{11}{26}\right)z + .05z}{.6} = \frac{2}{13}z$$

$$y = \frac{.275}{.65}z = \frac{11}{26}$$

$$z = z$$

$$\vec{x} = z\begin{pmatrix} 2/13 \\ 11/26 \\ 1 \end{pmatrix} \qquad z\left(2/13 + 11/26 + 26/26\right) = \left(\frac{41}{26}\right)z \qquad \frac{41}{26}z = 1 \qquad z = \frac{26}{41}$$

$$u^* = \left(\frac{26}{41}\right)\begin{pmatrix} 2/13 \\ 11/26 \\ 26/26 \end{pmatrix} = \begin{pmatrix} 4/41 \\ 11/41 \\ 26/41 \end{pmatrix}$$

sum = 1

a) If original guy is a Farmer $\vec{x}^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

his son's probability is $\vec{x}^{(1)} = M\,\vec{x}^{(0)} = \begin{pmatrix} .4 \\ .3 \\ .3 \end{pmatrix}$

grandson's probability is $\vec{x}^{(2)} = M\,\vec{x}^{(1)} = \begin{pmatrix} .205 \\ .285 \\ .51 \end{pmatrix} \begin{array}{l} F \\ BC \\ WC \end{array}$

$\therefore$ the grandson of a farmer will have .205 chance of also being a Farmer

b) In the long run, the proportion of male population that will be Farmers is $4/41 \simeq 10\%$

10.4.3

$$M = \begin{array}{c|cc} & \text{city} & \text{country} \\ \hline \text{city} & .95 & .15 \\ \text{country} & .05 & .85 \end{array}$$

$$M - I = \begin{bmatrix} -.05 & .15 \\ .05 & -.15 \end{bmatrix} \rightarrow \begin{bmatrix} -.05 & .15 \\ 0 & 0 \end{bmatrix} \qquad \begin{matrix} x = 3y \\ y = y \end{matrix}$$

$$\vec{x} = y\begin{pmatrix} 3 \\ 1 \end{pmatrix} \qquad \text{sum} \quad 4y = 1$$

$$y = \frac{1}{4}$$

$$u^* = \frac{1}{4}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{bmatrix} 3/4 \\ 1/4 \end{bmatrix} \begin{matrix} \text{city} \\ \text{country} \end{matrix}$$

sum = 1

a) $u^{(k+1)} = T\, u^{(k)}$

$$u^{(2003)} = \begin{bmatrix} 35\,000 \\ 25\,000 \end{bmatrix}$$

$$u^{2004} = \begin{bmatrix} .95 & .15 \\ .05 & .85 \end{bmatrix}\begin{bmatrix} 35{,}000 \\ 25{,}000 \end{bmatrix} = \begin{bmatrix} 37\,000 \\ 23\,000 \end{bmatrix} \begin{matrix} \text{city} \\ \text{country} \end{matrix}$$

$$u^{2005} = T\, u^{2004} = \begin{bmatrix} 38{,}600 \\ 21{,}400 \end{bmatrix}$$

$$u^{2006} = T\, u^{2005} = \begin{bmatrix} 39{,}880 \\ 20{,}120 \end{bmatrix}$$

$$u^{2007} = T\, u^{2006} = \begin{bmatrix} 40{,}904 \\ 19{,}096 \end{bmatrix}$$

$$u^{2008} = T\, u^{2007} = \begin{bmatrix} 41\,723.2 \\ 18{,}276.8 \end{bmatrix} \begin{matrix} \text{city} \\ \text{country} \end{matrix}$$

b) eventual population distribution:

city $= (3/4)\, 60000 = 45000$

country $= (1/4)\, 60000 = 15000$

10.4.7

$$T = \begin{pmatrix} .5 & .25 & 0 \\ .5 & .5 & .5 \\ 0 & .25 & .5 \end{pmatrix} \begin{matrix} R \\ P \\ W \end{matrix} \qquad T-I = \begin{pmatrix} -.5 & .25 & 0 \\ .5 & -.5 & .5 \\ 0 & .25 & -.5 \end{pmatrix} \xrightarrow{L_{21}=-1} \begin{pmatrix} -.5 & .25 & 0 \\ 0 & -.25 & .5 \\ 0 & .25 & -.5 \end{pmatrix} \to \left[\begin{array}{cc|c} -.5 & .25 & 0 \\ 0 & -.25 & .5 \\ 0 & 0 & 0 \end{array}\right]$$

$$-.25y = -.5z$$

$$y = 2$$

$$x = \frac{.25}{.5}y = \frac{1}{2}(2) = 1$$

$$\vec{x} = z\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$z(1+2+1) = 4z$$

$$4z = 1$$

$$z = .25$$

$$\vec{x} = .25\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} .25 \\ .5 \\ .25 \end{pmatrix}$$

↳ sum = 1

PINK DISTR.

$$\overset{M_{PINK}}{\begin{pmatrix} .5 & .5 & .5 \end{pmatrix}} \overset{\vec{x}}{\begin{pmatrix} .25 \\ .5 \\ .25 \end{pmatrix}} = .5$$

10.5.1

$$T = \begin{pmatrix} 1 & 1 \\ -1 & -\frac{7}{6} \end{pmatrix} \qquad P_A = (1-\lambda)(-\tfrac{7}{6}-\lambda)+1 = \lambda^2 + (\tfrac{1}{6})\lambda - (\tfrac{1}{6})$$

$$\frac{-\frac{1}{6} \pm \sqrt{\frac{1}{36} - 4(-\frac{1}{6})}}{2} = -\frac{1}{12} \pm \frac{\sqrt{25/36}}{2}$$

a) spectral radius = $|\lambda|_{\text{greatest}} = \frac{1}{2}$

$$-\frac{1}{12} \pm \frac{(\frac{5}{6})}{2} = -\frac{1}{12} \pm \frac{5}{12}$$

b) $\begin{pmatrix} 1 & 1 \\ -1 & -\frac{7}{6} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$$\boxed{\lambda_1 = \frac{1}{3} \quad \lambda_2 = -\frac{1}{2}}$$

exact solution:

$$\left(\begin{array}{cc|c} 1 & 1 & -1 \\ -1 & -\frac{7}{6} & 2 \end{array}\right) \rightarrow L_{21} = -1 \left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & -\frac{1}{6} & 1 \end{array}\right) \rightarrow \begin{array}{l} x = -1 - y \\ y = -6 \end{array} \quad x = -1 + 6 \quad \begin{array}{l} x = 5 \\ y = -6 \end{array}$$

$$\begin{pmatrix} 1 & 1 \\ -1 & -\frac{7}{6} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ -\frac{6}{7}\ \frac{6}{7} & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ -\frac{12}{7} \end{pmatrix}$$

$x = -1 - y$

$y = \frac{-12}{7} - \frac{6x}{7} = \frac{-12 - 6x}{7}$

☆ with 6 iterations the approximations are getting closer to the exact values. Since the error is so large it will take many more iterations to get very close to the exact values (above)

	$\vec{x}^1$	$\vec{x}^2$	$\vec{x}^3$	$\vec{x}^4$	$\vec{x}^5$	$\vec{x}^6$
x	-1	$5/7$	$-1/7$	$\frac{65}{49}$	$29/49$	$\frac{635}{343}$
y	$\frac{-12}{7}$	$-\frac{6}{7}$	$\frac{-114}{49}$	$\frac{-78}{49}$	$-\frac{978}{343}$	$-\frac{762}{343}$

10.5.5

a) $\begin{pmatrix}3 & -1\\ -1 & 5\end{pmatrix}$ $u=\begin{pmatrix}2\\1\end{pmatrix}$ exact: $\left(\begin{array}{cc|c}3 & -1 & 2\\ -1 & 5 & 1\end{array}\right)$ $L_{21}=-\frac{1}{3}$ $\left(\begin{array}{cc|c}3 & -1 & 2\\ 0 & 14/3 & 5/3\end{array}\right)$

$y=\frac{5}{14}\simeq .357$

$x=\dfrac{+\frac{5}{14}+2}{3}=\frac{11}{14}\simeq .786$

close to exact values

$\begin{matrix}\frac{1}{3}\\ \frac{1}{5}\end{matrix}\begin{pmatrix}1 & -1/3\\ -1 & 5\end{pmatrix}\begin{matrix}x\\ y\end{matrix}=\begin{pmatrix}2/3\\ 1\end{pmatrix}$

$x=\frac{2}{3}+\frac{1}{3}y$

$y=\dfrac{1+x}{5}$

	$\vec{x}^1$	$\vec{x}^2$	$\vec{x}^3$	$\vec{x}^4$	$\vec{x}^5$	$\vec{x}^6$	
x	$2/3$	1	$7/9$	$4/5$	$\frac{106}{135}$	$59/75$	$\simeq .786$
y	1	$1/3$	$2/5$	$16/45$	$9/25$	$\frac{241}{675}$	$\simeq .357$

10.5.5

d) $\begin{bmatrix}3 & -1 & 0\\ -1 & 2 & 1\\ 0 & 1 & 3\end{bmatrix}$ $u=\begin{pmatrix}1\\-1\\0\end{pmatrix}$

exact: $\left[\begin{array}{ccc|c}3 & -1 & 0 & 1\\ -1 & 2 & 1 & -1\\ 0 & 1 & 3 & 0\end{array}\right]\rightarrow L_{21}=-\frac{1}{3}\left[\begin{array}{ccc|c}3 & -1 & 0 & 1\\ 0 & 5/3 & 1 & -2/3\\ 0 & 1 & 3 & 0\end{array}\right]$

$L_{32}=3/5\left[\begin{array}{ccc|c}3 & -1 & 0 & 1\\ 0 & 5/3 & 1 & -2/3\\ 0 & 0 & 12/5 & 2/5\end{array}\right]$

$x=(1+(-1/2))/3=1/6$

$y=(-2/3-\frac{1}{6})/(5/3)=-1/2$

$z=\frac{1}{6}$

$x=1/6=.1667$

$y=-1/2=-.5$

$z=1/6\simeq .1667$

close to exact values

$\begin{matrix}\frac{1}{3}\\ \frac{1}{2}\\ \frac{1}{3}\end{matrix}\begin{bmatrix}1 & -1/3 & 0\\ -1/2 & 1 & 1/2\\ 0 & 1/3 & 1\end{bmatrix}\begin{bmatrix}1/3\\ -1/2\\ 0\end{bmatrix}$

$x=\frac{1+y}{3}$

$y=-\frac{1}{2}+\frac{x}{2}-\frac{z}{2}=\frac{x-z-1}{2}$

$z=\frac{-y}{3}$

$\vec{x}^1$	$\vec{x}^2$	$\vec{x}^3$	$\vec{x}^4$	$\vec{x}^5$	$\vec{x}^6$	$\vec{x}^7$	
$1/3$	$1/6$	$2/9$	$1/6$	$5/27$	$1/6$	$14/81$	$\simeq .173$
$-1/2$	$-1/3$	$-1/2$	$-4/9$	$-1/2$	$-13/27$	$-1/2$	$= -.5$
0	$1/6$	$1/9$	$1/6$	$4/27$	$1/6$	$13/81$	$\simeq .161$