Homework: Due 8-29
Math 513

Problems from textbook: §1.2 - 1.2, §1.3 - 1, 2, 6, §2.1 - 3, 4, 5,
Review problems: Matrix multiplication §1.5 1.2, 3 (Not to be turned in)

Question 1. Let $F$ be a field. Let $E_1: a_1x_1 + a_2x_2 + a_3x_3 = y_1$, $E_2: b_1x_1 + b_2x_2 + b_3x_3 = y_2$, $E_3: c_1x_1 + c_2x_2 + c_3x_3 = y_2$ be linear equations with coefficients in $F$. Let $d_1, d_2, d_3$ be scalars. Prove that any solution $(z_1, z_2, z_3)$ to the system of equations \{ $E_1, E_2, E_3$ \} is a solution to the equation $d_1E_1 + d_2E_2 + d_3E_3$.

Proof. Let $(z_1, z_2, z_3)$ be a simultaneous solution to $E_1, E_2$ and $E_3$. Then,

$$d_1(a_1z_1 + a_2z_2 + a_3z_3) + d_2(b_1z_1 + b_2z_2 + b_3z_3) + d_3(c_1z_1 + c_2z_2 + c_3z_3) = d_1y_1 + d_2y_2 + d_3y_3$$

and so $(z_1, z_2, z_3)$ is a solution to $d_1E_1 + d_2E_2 + d_3E_3$.

Question 2. Prove Theorem 1 in §1.2. You may use the following generalization of Question 1 without proving it: If $E_1, \ldots, E_k$ are a collection of linear equations in $n$-variables and $d_1, \ldots, d_k$ are scalars, then any solution to the system \{ $E_1, \ldots, E_k$ \} is also a solution to $d_1E_1 + d_2E_2 + \ldots + d_kE_k$.

Proof. Let $\{E_i\}_{i=1}^k$ and $\{E'_j\}_{j=1}^m$ be equivalent systems of linear equations. Let $z$ be an $n$-tuple which is a solution to the system $\{E_i\}_{i=1}^k$. For each $j$, $E'_j$ is a linear combination of the $\{E_i\}_{i=1}^k$ and so by the generalization of Question 1, $z$ is a solution to $E'_j$. Since this is true for all $j$, all solutions of the first system are solutions to the second system.

Similarly, by reversing the roles of $\{E_i\}_{i=1}^k$ and $\{E'_j\}_{j=1}^m$, all solutions to the second system are also solutions to the first system. Thus, they have the same set of solutions. \qed