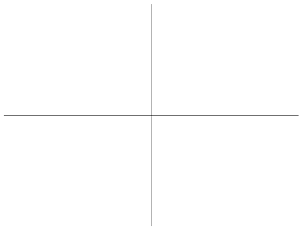
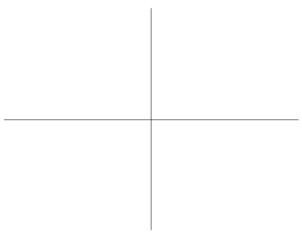
DIFFERENTIABILITY (2.6) NAME________________________________

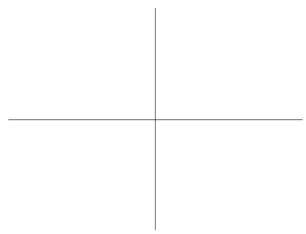
1. Determine if $f\,'(0)$ exists for $f(x) = \left(x + |x|\right)^2 + 3$. Include an accurate sketch.



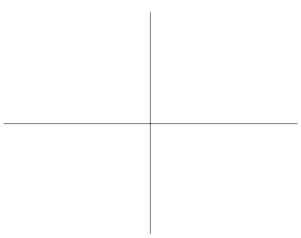
2. In each case, use the graph of $f(x)$ to sketch a graph of $f\,'(x)$. Label all important features of each graph. Hint: Each function has some hidden features that you might not see on the standard window.

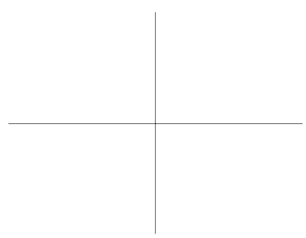
A. $f(x) = 3x^{2/3} - x$



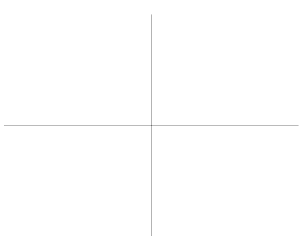


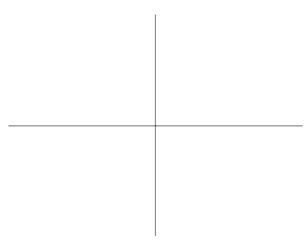
B. $f(x) = 3x^{1/3}\left(2 + x\right)$





C. $f(x) = 28 + |13 - x| + |5 - x|$

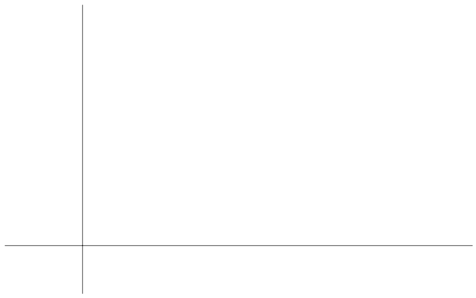




3. The acceleration due to gravity, g, is a function of the distance from the center of the Earth, r. Let R be the radius of the Earth, M be the mass of the Earth, and G be the gravitational constant.

$$g(r) = \begin{cases} \dfrac{GMr}{R^3} & r < R \\ \dfrac{GM}{r^2} & r \geq R \end{cases}$$

A. Sketch a graph of $g(r)$. Label all important features.



B. Is g a continuous function of r? Is g a differentiable function of r?

4. Find values for m and b so that $g(\theta)$ is differentiable at $\theta = 0$.

$$g(\theta) = \begin{cases} \sin(2\theta) & \theta \leq 0 \\ m\theta + b & \theta > 0 \end{cases}$$

5. Use the definition of the derivative to determine if $f\,'(0)$ exists in each case.

A. $f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$

B. $f(x) = \begin{cases} x \sin\left(\dfrac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$