

More on limits, continuity, and differentiability

1. Discuss the continuity of the function  $R(t) = t^2 \cdot e^{1/t}$ . In particular, if there is a discontinuity, determine if it is removable or essential and show algebraic work.

2. Determine if the following function is continuous at  $x = 1$ . If so, determine if it is differentiable at  $x = 1$ .

$$f(x) = \begin{cases} \ln(x) & x > 1 \\ (1.7)^x - 1.7 & x \leq 1 \end{cases}$$

3. Determine the values of  $A$  and  $B$  so that the function  $g(r)$  is continuous.

$$g(r) = \begin{cases} \frac{r \log(r^2)}{r+1} & r \neq 0, -1 \\ A & r = 0 \\ B & r = -1 \end{cases}$$

4. The analysis of blood flow through the heart leads to a function of the form

$$f(r) = -2|r| + \sqrt{1 - 4r^2 + 4|r|}.$$

A. Investigate the differentiability of  $f(r)$  at  $r = 0$  graphically.

B. Rewrite  $f(r)$  without absolute values.

C. Use your equation in part B to find the slope of  $f(r)$  for  $r > 0$  and for  $r < 0$ .

D. What do your answers to part C tell you about the differentiability of  $f(r)$  at  $r = 0$ ?