PARTICIPATION IN THE MATHEMATICS CLASSROOM: DOES EVERY STUDENT HAVE A VOICE?

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This article addresses the concept of participation in the mathematics classroom, especially as it relates to students from certain ethnic and language groups and economically disadvantaged students. We are primarily concerned with seeking ways to develop approaches to mathematics education that are sensitive to the contexts and lived experiences of all learners.

In particular, we are concerned by how to develop classroom-teaching environments that promote the mathematical participation of *all* students. In our local contexts we work with students who are often left behind in the academic journey. We present two cases located in geographically very distant settings. They share essential characteristics, however, that point to problematic areas that transcend borders, especially in reference to how certain groups of students, such as immigrants, members of specific ethnic and language groups and economically underprivileged students, are viewed by the educational systems in place. These two cases illustrate the effects of social and organizational structures on students' participation in the mathematics classroom.

Our work is grounded on a sociocultural view of education (Moll, 1992; Nunes and Bryant, 1996; Lerman, 2001). Sociocultural theories emerged as an alternative to a perceived dichotomy between social and the psychological or individual perspectives. In the psychological or individual approach, the notion of participation is centered on the learner and pays little attention to the characteristics of the learning context. In the social approach, the key notion of participation is viewed as a kind of socialization into the mathematical practices. The participation model, as understood in the sociocultural approach, focuses on the use of discourse and some of its contents (norms, values, valorizations) as crucial mediating tools in order to interpret the mathematical learner in context. The acquisition of concepts and skills is not enough in the process of becoming a mathematical learner. There also needs to be an active participation in the reconstruction of a specific kind of discourse.

In this article we focus on barriers to this active participation. Common to both cases is our (including the teachers') focus on the experiences that each student brings to the mathematics classroom. So, rather than looking at certain students from a deficit-model approach that focuses on what they are lacking, we question the educational policies that seem to lead to these barriers. Although the two cases are located within experiences in which the teachers attempted to change the nature of participation, in this article our focus is not on the nature of those attempts but on the situation that prompted the teachers to try to change the patterns of participations. Specifically, we illustrate how the students themselves are aware of the social and organizational structures in place and of the effect of these on their participation.

Case 1: "How come I don't get to be in GATE?"

This case [1] takes place in a fifth grade classroom (ten-year-old students) in a bilingual school (English and Spanish) in Tucson, Arizona. By the time they reach fifth grade, many of the students at this school have been together for up to five years and have developed strong bonds of friendships as well as rivalries. The twenty-nine children (fourteen boys and fifteen girls) in the class in which the study took place came from predominantly working class families. The class was comprised of nineteen children who were Mexican American or of other Hispanic origin, five who were Anglo American, four who were African American, and one native American.

The fifth grade teacher was a teacher-researcher in one of our research projects. She was open to try approaches that would reach out to her students while helping them develop a strong mathematical basis for middle school. To that end, we developed a series of learning experiences, such as a learning module centered on games and another centered on the geometry of our local context. First of all we had to work on developing social and sociomathematical norms (Yackel and Cobb, 1996) in the classroom that were quite different from what the students had experienced in previous years. Mathematics was not a subject that they were used to talking about. For these students, to do mathematics meant to fill out a worksheet and return it to the teacher for evaluation. The idea of discussing a problem, listening to different points of view and building on each other's ideas was not part of their experience as mathematical learners (see Civil, 2002, for more details on the work in this classroom).

When we tried to open up the patterns of participation in the classroom, the power and status structures were deeply engrained. There were two groups who had high status in this class and therefore exerted power: those who were good at certain sports (primarily basketball and American football for the boys and, to a certain extent, softball for the girls) and those who were in the Gifted and Talented Education (GATE) program (seven students, four of whom were Anglo American). In this article we focus on the obstacles we encountered as we tried to bring about change and to move towards the participation of all students. We want to draw attention to how powerful certain structures, such as GATE and the sports program, are in determining whose participation is going to be valued. We do this by focusing mostly on students' voices and behaviors, to show how aware these ten-year-old students were of "how the system works".

Being good at certain sports was a defining characteristic of popular students. At the beginning of the year we conducted interviews with all the students in order to establish a rapport with them. Most of the boys expressed an interest

in sports and many of them declared that becoming a basketball or an American football player was their career goal. In one of the mathematics projects, the students had to work in groups and develop their own games. They had to write the rules for their game, build it, and show how mathematics was being used in their games. If the games were sports-related or came from students who were in the popular group, they were well received.

One of the games, created mainly by Brandon [2] (a highly proficient basketball player who was in GATE), centered on the theme of basketball. It was a game for two players, each of whom had to move a basketball-player counter along a number line numbered from 2 to 12. In order for players to move, they had to roll the corresponding sum on two dice. This had to be done in numerical order. So, first a player had to roll a sum of 2, then a sum of 3, then a sum of 4 and so on. Hence, for a player to move to the first position (2), the player had to roll a one on both dice, something that students soon found out could take a long time. This game was closely related to probability explorations that we had just done in class, yet this connection did not appear either in the students' comments and questions during the presentation of the game, nor in their written anonymous evaluations.

During the presentation of their game, in an attempt to get to the mathematics behind it, the first author suggested that two students play it for a while. Two boys came to the front of the class and started rolling the dice (taking turns) looking for the necessary two to be able to move to the first position. As they kept rolling the dice and getting other numbers, Brandon said:

> It's hard to get a two, when you get to the higher numbers, it's easier.

The teacher then asked a clarification question:

> So, the first person to get a two is the one who will move his player?

to which Andrew (one of the boys playing the game) said:

> If that ever happens.

The boys persisted for quite a while, rolling the dice. Eventually, Brandon and his partner suggested trying the game with two other students. All throughout their presentation, the teacher and the first author interjected questions and comments to try to get the students' attention to the mathematics behind the game. Our attempts to engage the class in a critique of this game from a mathematical point of view – connecting it to the previous work on the probabilities of getting the sums of 2 through 12 when rolling two dice – fell flat.

In their written comments on the game, only one comment reflected an awareness of how hard it was to get these totals in order. Most comments were supportive of the game based on the fact that it was about basketball:

> I liked this game because it was about basketball.
>
> I liked Brandon's game because they presented the game I like, basketball, so I liked their game.
>
> [It] was good because it had basketball.

We do not know whether students were aware of the mathematical reasons for why, as Brandon said, "it's hard to get a two". As the teacher pointed out several times during our work in her classroom, these students were not used to analyzing each other's work, especially within a mathematics context (Civil, 2002). What we do know is that Brandon was one of the most popular students in the class. This, combined with his choice of a game centered on basketball, probably took precedence in students' minds as they 'evaluated' the game.

To change the patterns of participation may be very difficult when certain students have high status and their contributions are valued for who they are and not for the content, while other voices are not heard. In the mathematics class, not all GATE students were well liked, but when they spoke, students tended to side with them. In an end of the year interview, two of the Anglo American girls who were in GATE and who stood out in class for their academic achievements reflected on their experiences in this class. They were aware about how the rest of the class felt about them:

> Every time I raise my hand, everybody goes very quiet to see if I get it wrong; because if I do, they go, "Eeey, Rebecca got it wrong, she's probably going to go crying".

When probed about why it was that they did not participate as much as we would have expected, Rebecca said:

> I like to show off my knowledge but when I do they always tease me.

At the beginning of the year, Rebecca had already expressed her concern at "how mean the children in this class were":

> A lot of people tease me and fight here. At my old school [where she was in an all-day GATE class], to make you feel bad they would tell you that you're not good at math. Here some people go out of their way to be mean.

The two girls, Rebecca and Melanie, had carefully studied the classroom dynamics. Rebecca's group game was a board game that represented an ethnography of the class from these students' point of view. The players moved around the board (the classroom) and the game cards that told them what to do were all representative of behaviors and events that had taken place in this class. These two students were aware of how their participation in the mathematics classroom affected others' participation. They both had noticed that when Melanie gave an answer, most of the boys quickly sided with her and went along with the answer. When Rebecca gave out an answer, it was mostly the girls who sided with her. They had also noticed that:

> M: They don't act up as much if I get something wrong as if Rebecca does.
>
> R: Because Melanie has been here longer.
>
> M: Yeah, I've been around more.

These two students were aware of the role that sports played in this school:

M: Sports gives them [other students in the class] an advantage over Rebecca. They are constantly seeing what she is going to miss.

Classroom dynamics can be affected by having these kinds of subgroups, whether it is the popular children who are good at sports, or the students who get pulled out for GATE, or those who are pulled out for special education (two boys in this class, one Hispanic and one native American). In one of the problem-solving activities, students worked in groups on how to write the numbers one through thirty-five as sums of consecutive numbers. After they had worked on this problem, we asked each of them to reflect in writing on their experience of working in their group. Two of the answers from one group caught our attention. They came from Rebecca and Marcos (a Hispanic student who had been labeled as having a learning disability and was pulled out for special education attention on a regular basis).

Rebecca felt frustrated because there was only one person (not Marcos) in her group who made some attempt at helping her. She tried to get people to help but "it didn't work", she wrote. Marcos, in response to the question, "what didn't you enjoy [about the work in his group]?" wrote, "that Rebecca did everything". Marcos was a quiet student, not particularly successful by school standards, and who often tended to take a passive role (this notion of passive role is also present in the next case). Yet, in this instance, he seemed to be claiming the right to be noticed as an active player, while at the same time, Rebecca was complaining that her group did not want to help. We wonder if they were aware of each other's perceptions.

As the year went on, Marcos and other students did begin to participate more in the discussions in the mathematics class, but the fact remained that as the discussions became more mathematical, those students who were in GATE dominated the discussions. Was it because the mere fact of being in GATE gave them authority when speaking about academic matters? Or was it because the GATE experiences prepared them better for the kind of mathematical discussions that we were having? Probably both, but we are concerned that students who are placed in the GATE program (based primarily on their achievement on specific tests) are actually given more challenging activities in the pull-out time, thus widening the gap with the rest of the students who do not have access to these opportunities. Andrew's case shows that children are aware of this irony.
Andrew, a Hispanic student, wanted to be a basketball player and was very good at this sport. He was also remarkably talented for mathematics though he tended to act up and be somewhat disruptive. The teacher thought this was because he was very intelligent and was bored in class. As the year went on, we saw Andrew's thinking in action. In a group task on finding the angles of different polygons, he was the only one who persisted and reasoned through to find every single angle in the different polygons. In one of the parallelograms, they had already determined that two of the angles were 30° each. For the other two angles, they first came up with 120° for each. The group was ready to leave the problem at this. They had been working on this task for quite some time and they were getting restless. Then Andrew realized that 120 + 120 + 30 + 30 was 300. He said, "We need 60 more, so it's 150 and not 120." He sounded confident, as if everything they had been doing with the measurement of angles had suddenly fallen into place.

Throughout the year we had many opportunities to see Andrew as being extremely talented in mathematics. If a problem caught his interest (such as finding the angles of the different polygons, or when working in Logo, making the turtle draw a basketball court to scale) he would persist at it and often succeed. He was also argumentative and sometimes he used this skill to engage in mathematical arguments, such as when he started challenging other students to explain their reasoning. He would often be ready to challenge his peers' explanations with a "why" question, or with a counterexample. He was unpredictable, though, and maybe that is why he did not make it into GATE (as the teacher explained, in addition to the test results, children have to show that they are "ready").

One day, as the seven GATE students left the classroom to go to their special activities, Andrew said:

> If GATE is to make us more intelligent, how come I don't get to be in GATE so that I can get smart?

The teacher made some kind of general comment, but Andrew's poignant question was not addressed. We saw how perceptive students were about where everybody stood in class, and who was valued when. Rebecca's reflection on the GATE system gives further indication of students' awareness about how the system works:

> GATE tends to be upper class white people, I've noticed, it's kind of a corrupt system.

This case illustrates some of the barriers that we encountered (and that the students themselves encountered) as we tried to open up the patterns of participation in the mathematical classroom. The popular students (usually those in a school sports team) tended to be listened to. In more academic tasks, the GATE children tended to dominate and were clearly the leaders. We turn now to the next case, where again our focus is on the students' perspective to show how aware they are of who has a voice and in what situations.

Case 2: "We are all 'Sleeping Beauties' in this room!"

The second case is located in Barcelona, Spain. It is part of a larger research project [3] that has as a primary goal to find more appropriate ways to teach mathematics to students socially at risk and with a clear lack of power. After investigating the ways in which the educational administration organizes the schooling for these students, we found constraints at the educational level derived from decisions at the political level. In order to ensure that "all" students achieve the standards in mathematics, a parallel system of special classrooms is created within the regular system in secondary schools that are highly multiethnic.

In this article we look at the implementation of such a system in a classroom of sixteen-year-old students. In this classroom, out of twenty-seven students, there were six special needs students [4] who went to a different room as part of the compensatory program that involves reading, writing

and mathematics. These six students spent three hours a week in an individualized learning program with a mathematics teacher. In the regular classroom, the twenty-one mainstream students received mathematics instruction characterized by working in small groups on a variety of problem-solving tasks. All twenty-seven students met together for one fourth hour of mathematics per week, in which the regular class teacher continued her teaching routine of a co-operative learning approach. The data coming from the interactions between students in the general system and those in the special needs system help us to analyze participation difficulties for some students.

What determines who is a 'special needs' student? There are two main factors: the level of proficiency in the official language(s) (Catalan or Spanish) and discipline issues. Because most immigrant students, particularly recent arrivals, are often not quite proficient in their command of Catalan or Spanish, they are placed in the special needs category. And among the students labeled as 'disruptive', many of them were living in high-poverty situations. Therefore, students with special needs are characterized by poverty, limited proficiency in the official language, and immigrant group status. Cognitive factors and knowledge of mathematics do not seem to play a role in determining whether a student is a special needs case. The placement in special classes is considered as a way to help them adjust to the norms and correct their tendency to 'inappropriate' forms of behavior according to the dominant point of view. Being a candidate for special mathematics education has to do then with the students' distance from the legitimate norms in the regular classroom.

The twenty boys and seven girls in the classroom (four boys and two girls in the special needs program) have very different cultural backgrounds and have been together for the last eight months. As in the first case, we focus on what happened when these students were put in situations where there was an attempt to open up the patterns of participation. That is, we look at what happened during the social interactions in the fourth hour of mathematics when all students, both the special needs and the mainstream ones, were put together and asked to collaborate with each other. The class was clearly divided into two groups with very different status and with some individual differences within each group. Those who were in the special needs program were not allowed to participate in the same way as those who were always in the regular class. Moreover, most special needs students identified themselves with certain forms of participation and were not willing to interfere in the pedagogical discourse of the classroom because they felt it was not convenient for the right dynamics of the mathematics lesson. Isabel, a Gypsy student, clearly distinguished what she could do in the regular class from what she could do in the special class. She thought that a passive attitude was more appropriate in the regular class in order not to confuse other students:

> I do not want to make people feel confused. If I do not understand what they say in the regular class, I wait and if I remember I'll ask the teacher in the special class [...] sometimes I forget what I wanted to ask! [5]

Some of these students behaved differently depending on whether they were in the regular mathematics class (the fourth hour) or in the special needs program. For example, Lidia, another Gypsy student, expressed a shy attitude during the fourth hour, which contrasted with the active and open attitude during the other three hours with the special needs teacher:

> *On Monday, in the special class*: Hey, Miss, I'm not sure I have done it correctly, can you please check it before I go on with the next exercise?
>
> *On Thursday, in the regular class*: Wh …wh …what? Yes, Miss … Ummm … I prefer not to … If you … Yes, Miss [when asked to comment on the problem].

It could be argued that her changing attitude was due to the different role played by the two teachers, the regular one and the one in charge of special needs education, or by the number of students in each case, twenty-seven *vs* six. When we asked Lidia, "do you feel comfortable during the regular classes with the whole group?", she answered:

> I'm always very nervous because I'm not used to deciding everything … my peers are very hard on me, I cannot do it and they wait for me and start to put me down … and I don't know what to decide because they don't explain it to me … all of them are always in this class and I'm not! When they call me a foreigner … I cannot stand it. I'm not a foreigner because I want to, but because teachers let it happen.

Lidia reminds us of Rebecca in the first case. They both feel threatened by their peers. But Rebecca exudes confidence, probably because she is academically very successful. For Lidia the situation is harder as she feels (or is made to feel) like an outsider. Lidia is not used to working in small groups, yet, on that fourth hour she is expected to do so. But what seems more uncomfortable to her is the attitude she notices from the other members in her group. None of them is a special needs student and she thinks that they treat her differently because she is one. Her group's discussion appears to be full of references to her "special needs" label. The use of "foreigner" as a nickname is an important signal of the low status that Lidia sustains in the regular class. The regular class students used the term "foreigner" to refer to students who were in the special needs class and who did not speak Catalan well. Upon being asked for more details about this nickname, Lidia revealed that:

> They always call us foreigners when they want us to feel bad, they know we don't like it, but they still go on calling us foreigners … well, they don't call Manuel a foreigner, though he is, because they like him, he is very good at soccer … the teacher doesn't like them to call us like that either … he gets angry … I prefer to be called a Gypsy rather than a foreigner … you know, they do not prefer Manuel because he is better at mathematics, it is only because of soccer.

There are many issues involved in the construction of status in the mathematics classroom. In Barcelona, as in the Tucson case, to be good at sports makes status increase.

Manuel and Lidia are both given a low status because of their membership of the special needs class and probably because of their sociocultural identity as Gypsies, but Manuel is more accepted than Lidia because of his ability at sports. These two students have many issues in common related to their position in the classroom. They have to cope with norms that contradict those they are used to. They were both shocked when the regular teacher tried to engage them in collaborative tasks. They felt it was absurd to share ideas and strategies to solve a mathematical problem with "non-experts". They also have in common their continuous references to their social identity, as Manuel explained in an interview, in reference to the use of collaborative tasks:

> I must look out for myself! If I start to worry about my peers I'm lost! The place where I live is very dangerous and you must look out for yourself, you only can count on your family, and they [the other students] are not my family. With the other teacher we don't waste our time talking and talking, each of us works on our worksheet and each of us gets the grade we deserve.

Arundhati also illustrates the obstacles in participation that special needs students confront. Arundhati is originally from Karachi, Pakistan, and she came to Barcelona when she was six years old. She referred to her situation in the mathematics classroom using the metaphor of someone sleeping. She had strongly internalized a passive role:

> It is like that tale, the tale of the Sleeping Beauty, did they explain it to you? We do not know enough mathematics and we must be in a different room, but some day we will wake up and we will know more mathematics and then we will be in the other room. We are all Sleeping Beauties in this room! And if we stay in this room it will be easier to learn all the mathematics that we do not know.

Arundhati explained the reasons why she should not be accepted by her peers. She also made explicit her conviction that when she finally became accepted it would make a great difference in her forms of participation. When she was asked, "when will you wake up from being a Sleeping Beauty?", she said:

> Someday I will be with the whole class and my peers will like my ideas. Now my ideas are not good for them because they know more mathematics and they know how to solve the problems. Maybe next year I will fit in this class […]. Next year I will say more things because I will know how to explain things better.

It is as if these special needs students had accepted their passive role, whether it was so as not to confuse the other students (Isabel and Lidia), or because the co-operative model did not match their reality outside school (Manuel), or because they do not feel quite ready yet for the mainstream group (Arundhati).

The perspective of the mainstream students gives a clear idea of how status is distributed in the classroom. The differences in the understanding of norms are one of the main issues used by the mainstream students to justify the low level of participation of their peers. Special needs students are seen as people who do not know how to behave and not necessarily as people who do not know enough mathematics. They are also labeled as students who are always trying to introduce issues from their private lives to defend their arguments. For the mainstream students, the use of personal experience in the mathematics classroom is a sign of inappropriate forms of behavior. Albert, a local student who never wanted to collaborate with any special needs student, stated:

> They [special needs students] have to learn how to behave in class […]. Most of them are always talking about their life. When they join us, they are always asking things, and when you explain the problem to them they always keep on refusing to accept it until the teacher comes to the group. Sometimes they have a right solution to the problem but it's useless because they cross it out. They never volunteer even if they have the right solution. They are a complete disaster!

Albert knew that he could lose status by addressing the wrong people in the classroom:

> We cannot say "Oooh, the poor little girl [referring to Lidia]! We must help her". We waste our time and she is not even going to understand it. My father doesn't like me to work with them. It is not that I'm forbidden to work with them but he prefers me not to waste my time.

Carmen, a local student considered to be good at mathematics, was very surprised when one day she found out that Lidia had solved a difficult problem posed to the whole class. She was not completely sure whether the teacher had said it was right with the purpose of motivating Lidia or because it was actually right:

> We had all solved it with numbers and the teacher said Lidia had a visual strategy that was very good. She had drawn a diagram to prove that a chessboard has 204 squares in total. I think the teacher wanted to encourage her because she is always very insecure. For me, the diagram was a mess, but that's the only thing she could do because she is not good at numbers. You know, she is in the special class.

Carmen had realized that some special needs students liked to introduce their personal experiences into the mathematical discussions, and that they often asked how the mathematics they were learning could be applied in real life. Carmen found it an inappropriate behavior and a way to hide their lack of mathematical competency. She was convinced that the use of real contexts in the mathematics classroom is a mistake:

> They are always asking "what does it mean?" They want the teacher to teach easier mathematics, but mathematics is very difficult, you cannot always find out what everything means […]. I don't blame them, when they are completely lost, they start talking about something that happened to their siblings and they expect us to believe that it has to do with the problem […]. They cannot expect everybody to be talking about their siblings in the math class … sometimes they are very funny!

Forms of participation linked with the introduction of real contexts are accorded low status by the mainstream students. Special needs students are scarcely valued mainly because of their placement in a different room, their social identity and also their use of daily experiences coming from outside the school. These students have internalized a powerless role in the mathematics classroom. Regular students in this case are considered in a similar way as GATE students in the Tucson case. The different weights that the different voices are given seem to be at the root of the forms of participation that take place in the mathematics classroom. The limits to the participation of all students are well illustrated by the Sleeping Beauty metaphor. When some of the sleeping students try to wake up by introducing personal experiences into the mathematical discussion, as in the Barcelona case, or by wanting to contribute, as in the Tucson case with Marcos, their participation is not acknowledged. In a sense, they are told to go on sleeping.

Reflection

The two cases deal directly with issues of participation in the mathematics classroom and more specifically with how different forms of participation seem to be strongly influenced by organizational structures at school and the emerging memberships that this creates. The internalization of certain roles, derived from these memberships, certainly has many implications for learning. The value of what is said and of who says it is established according to the place where each participant is located. Students respond according to where they are supposed to belong. Students placed in the high-status system may have easier access to the mathematical discourse, whereas students placed in the low-status system are still supposed to prove their value. Roles influence the students so that they learn to act and behave in ways that agree with the social order of the educational arrangements.

Whether it is overt tracking, the assigning of students to different groups or classes usually based on ability, or a somewhat more subtle approach by which certain students are given special attention at certain times, the fact is that different and thus differentiating curricula and pedagogical approaches are widespread and certainly not unique to the two countries in which the research presented here took place. Zevenbergen's (2003) study of students' perceptions of ability grouping in Australian schools relates very closely to our experiences in Tucson and Barcelona. The students' awareness of how the system works and of its consequences on their learning opportunities in the three cases (Australia, Barcelona, Tucson) is sobering. Zevenbergen's high-group-students' comments on being able to move quickly and not being slowed down by the "dumb kids" reminds us of Albert in the Barcelona case, and his concern for not wasting time with the special needs students. Zevenbergen's low- group-students' frustration with not being taught the hard questions reminds us of Andrew in the Tucson case and his poignant question, "How come I don't get to be in GATE?"

The decision, at the educational policy level, to offer different programs (and thus different learning opportunities) is often made in the name of meeting the needs of all students. The forms that these programs take, and the criteria by which students are placed in different programs, are likely to vary from context to context. The effectiveness of schooling seems to be associated with the classification of students into certain groups, and the mathematical discourses seem to be developed depending on this classification. If we look at the criteria for such classification, we can observe some presupposed mathematical abilities, linguistics competency or appropriate behaviors that justify the grouping. Mathematical ability, language and behavior are then behind the actions constraining patterns of participation.

In the Barcelona case, the grouping criteria were based on language proficiency and on behavior. In the Tucson case, the criteria appear to be more cognitively based, but one would have to question these criteria given the case of Andrew. It is important to note that, whatever the criteria, the fact is that in both cases the students who seem most negatively affected by these decisions are usually members of certain ethnic and language groups and are economically underprivileged. As Zevenbergen (2003) writes:

> […] ability grouping, therefore, reflects social rather than innate categories. (p. 5)

In the case of GATE in the USA, Ginorio and Huston (2001) write:

> Hispanics generally are underrepresented in Gifted and Talented Education (GATE) programs relative to their representation in the school population and in comparison to their White and Asian American peers. Hispanics are 14 percent of students nationwide, for example, and 8 percent of the GATE students […]. In Arizona, they are 28 percent of students and 11 percent of GATE students. (p. 5)

Thus, we worry that special programs are actually helping maintain (and maybe exacerbate) the differences in the society at large. In the case of GATE, parents can actually request that their children be tested for the program. But we wonder, how many low-income parents or from certain ethnic and language groups are likely to do that? Along the same lines, we wonder how many of the parents of the immigrant or Gypsy students in the Barcelona case are likely to question why their children are being placed in special needs classes? There is an implicit message that states that these kinds of questions do not adjust to the established norms and that they are not under negotiation.

In both cases the teachers used pedagogical strategies that encouraged students' participation by altering the traditional ways of teaching. Their inclusive practices, however, were faced with several barriers. These barriers may be interpreted in terms of learning obstacles and social exclusion. Although the notion of learning obstacles has not been directly treated in this article, it is clearly suggested by some of the barriers to the full participation that have been identified in our analysis of the two cases. In this article, learning obstacles are not related to students' preconceptions and misconceptions of some mathematical ideas, but to forms of excluding some groups from mathematics education such as certain organizational structures at school. Learning obstacles are a social phenomenon here within the micro-context of the multicultural mathematics classroom related to barriers that go much deeper than classroom dynamics.

Our interpretation is not far away from the sociopolitical one developed by Skovsmose (2000). On the other hand, the notion of social exclusion is also suggested by our data. The two cases illustrate important issues concerning the potential role of mathematics classrooms in reproducing wider societal patterns of advantage and disadvantage.

Notes

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[2] All students' names are pseudonyms.

[3] This research was funded by Fundació Propedagògic, a Catalan private foundation. The views expressed here are those of the authors and do not necessarily reflect the views of the funding agency.

[4] The original Catalan term used for 'special needs students' is 'alumnes amb necessitats educatives especials – NEE –'.

[5] The quotations from students were originally in Catalan except for the quotations from Gypsy students that were in Spanish. All quotations were translated into English by the second author.

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