WORKING TOWARDS EQUITY IN MATHEMATICS EDUCATION: A FOCUS ON LEARNERS, TEACHERS, AND PARENTS

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This paper presents a reflection on my research largely grounded on my interest in students', teachers', and parents' ideas about mathematics. Starting with some considerations from a cognitive point of view, in particular preservice teachers' understanding and beliefs, I move onto sociocultural aspects. I specifically address issues related to context, valorization of knowledge, participation, and in-school and out-of-school mathematics. I draw on examples from my research in Latino, working-class communities to highlight the need (yet the complexity) to focus on all interested parties (parents, teachers, and students) and on mathematics if we are to address equity in mathematics education.

In this essay[1] I reflect on my trajectory as a researcher in mathematics education, with an eye on the theme for this conference – focus on learners, focus on teachers. My entry into the world of mathematics education research was largely focused on teachers, and more specifically on preservice elementary teachers. As a researcher, my approach was essentially cognitive—I wanted to understand their understanding and to learn about their beliefs about mathematics. I was and continue to be fascinated by how people (teachers, children, parents) make sense out of mathematics and what role their beliefs play in this process. As a teacher educator, however, I wondered about the implications of preservice teachers' understanding of mathematics and their beliefs about its teaching and learning for the children they would be teaching (Civil, 1993). I was also concerned about how preservice elementary teachers were sometimes portrayed in a negative way, focusing on their inadequate understanding of mathematics. To me, these "inadequacies" were intriguing and, while a cognitive approach was certainly very helpful, the ideas of situated cognition and social and cultural context added to my understanding of those "inadequacies." Although equity per se was not in my agenda yet, I think that some of those initial experiences opened the way towards my interest in equity in mathematics education. A concern for those who are being left out of the mathematical journey seems to guide my work. Sometimes I wonder if I have moved away from my initial cognitive-based interest in research in mathematics education to address issues that focus largely on the social and cultural context, with mathematics playing a very peripheral role. As I look over my writing from the last few years, I notice that I often raise the question "where is the mathematics?" Mathematics plays a central role in my work and recently, in our current project, I find myself pushing for the mathematics in our activities and research discussions. My interest is in equity in mathematics education, where equity to me is related to access by **all** students to opportunities to engage in rich mathematics. In this paper, my goal is to share some examples from my research throughout the years, to illustrate the role of and the need for different frameworks in mathematics education research and in particular to argue for the need to combine cognitive approaches with sociocultural ones (Brenner, 1998; Cobb & Yackel, 1996). In doing so, I also aim to emphasize the need for a serious look at what we mean by equity. The word "equity" (or references along those lines) is present in most mathematics education documents (not only in the U.S., but based on my research collaborations with a colleague in Spain and conversations with researchers

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elsewhere in the world, it seems to be a widespread, relatively recent phenomenon), yet what do we mean by equity in mathematics education? Personally, it is hard for me now to look at any mathematics education area without an equity lens. For example, I have always had an interest in how students communicate about mathematics. Yet, my interest in this topic has considerably changed over the years. In Civil (1998) I focused on issues related to communication when students are working in small groups. My approach then was essentially cognitive, as I was primarily interested in the interplay of understanding and beliefs in small group discussions. More recently, my interest in communication relates to questions of participation (Civil & Planas, 2004): who has a voice in classrooms' discussions and whose voices are being heard. Yet, as I reflect on this more recent work and look at my current work, I am looking for how to frame the discussion of participation in such a way that mathematics becomes more central.

Research in Teacher Education: From Beliefs and Understanding to Equity

My first experience presenting at a conference was actually at PME-NA in 1989 (Civil, 1989). In that piece, a group of preservice elementary teachers were given a proportional reasoning task in which a fifth grader used incorrect reasoning (an additive approach) but the answer he obtained happened to be the correct one (at least in terms of a typical school mathematics task). The preservice teachers were to comment on this child's work. My emphasis in that paper was on questions such as "how ready are these prospective teachers to understand children's work. How are they going to handle it when one of their students comes up with a method different from theirs? What means do they have to determine the validity of a method?" (p. 292). I expressed concern for what I saw as a tendency to praise children's work without attention to the mathematics behind that thinking (Civil, 1993). Years later, I continued to express this concern, when I visited "reform" oriented classrooms in which children were encouraged to work in groups, discuss mathematics, look for different approaches to solve a problem, in short, many of the features that I value in a mathematical community of learners. But I also noticed how hard it is to listen to children's ideas about mathematics and what to do with that listening. As a result, I often heard comments along the lines of "great thinking" (was it always "great"?) and "thank you for sharing" (with no further discussion on the mathematical contribution of that sharing). Working on understanding how others (in most cases, students) make sense out of mathematics is one of the main reasons why I went into mathematics education. Whether I am working with children, preservice teachers, practicing teachers, parents, listening to their ideas about a mathematical situation fascinates me. Where is "equity" in this? When working with preservice teachers (and later on, with practicing teachers), I think I had an implicit concern for equity in that I worried about how a fragile understanding of the mathematics, and in particular of the mathematics for teaching (Adler & Davis, 2006; Ball & Bass, 2000) would affect their teaching and therefore their students' learning and enjoyment of mathematics. But that was the extent of my concern for equity at that time. In fact, I am not even aware that the term "equity" entered my conversation. In this paper I look at some of my work from those years with my current lens of equity. A typical topic in courses for prospective elementary teachers is a discussion of different algorithms for arithmetic operations. For example, one of the tasks I gave to that same group of preservice teachers to discuss was the "European" subtraction algorithm (the equal addition algorithm). I presented it to them as the way I learned how to subtract and they were to try to make sense out of it. Although my analysis of their discussion focused on cognitive aspects (e.g., assimilation to borrowing), some of their comments can certainly be looked at from a different angle:

Ann: Could you imagine if they said, "let's do math this way in American schools"?
Carol: Oh, my God!
Vicky: I don't think the kids would have as much problem with this as the teachers.
Ann: Uh, uh, you're right; that's exactly what would happen.
Carol: What's the value though? I mean, why are we doing this?

When talking about yet another algorithm for subtraction, in which the child had used negative numbers to find the answer (e.g., to do 62 – 48: 2- 8 = -6; 60 – 40 = 20; -6 + 20 = 14), Vicky said, "I do believe that you could eventually convince him that learning to carry is easier and leaves less room for error." And when talking about a left to right algorithm for subtraction, Carol said, " Wouldn't kids get confused? From left to right, wouldn't kids get confused? If I sat down with a group of kids and said, 'Ok, this is how you do it,' and showed them from left to right, I would think that when you got to the real thing, that they would get upset or that they would be confused."

Scenarios like the ones I just briefly presented can be analyzed from an understanding / cognitive approach: how do these preservice teachers understand these different algorithms? They can also be analyzed from a beliefs approach: what do they tell us about these prospective teachers beliefs about the teaching and learning of mathematics (as well as about their own beliefs about mathematics in general)? But these scenarios can also be analyzed from an equity point of view. For example, what are the implications of Ann's comment, "Could you imagine if they said, 'let's do math this way in American schools'?" or Carol's comment, "what's the value though? I mean, why are we doing this?" or Vicky's comment: "you could eventually convince him that learning to carry is easier" or Carol's comment "when you got to the real thing." What is the real thing? Is there a way (as in only one) that should be taught in "American" schools? And in the case of subtraction, is "the way" that of learning to "carry"? Is this why Carol wonders about the value of engaging in these discussions around different algorithms? My current research is located in low-income neighborhoods, with a large number of immigrant families—mostly from Mexico and Central America. Of particular concern to me is whether we are preparing teachers to address different approaches, particularly when those different approaches may be coming from low-income, immigrant children. About four years ago, I asked a class of preservice elementary teachers to write a reaction paper to an article by Perkins and Flores (2002) on the "mathematical notations and procedures of recent immigrant students." A few of the preservice teachers wrote comments indicating the need for immigrant students to learn the way arithmetic is done in the U.S. As one of them wrote, "this is nice but they need to learn to do things the U.S. way." Is it that they were concerned about their own understanding of these different ways, as one of the preservice teachers hints in the comment, "how can we be expected to know all these different ways?" Or is it related to valorization of knowledge (as in one way being better than the other) (or as in Carol's comment earlier of "what's the value though?").

With the rapidly changing demographics in the U.S., most teachers are likely to be in classrooms where children or their parents may have different approaches to doing mathematics. How do we incorporate or build on these approaches? What value do we give to the different approaches? The notion of valorization of knowledge is very present in my work, as it relates not only to my concern for equity but also to my other area of research on in-school / out-of-school mathematics. The next section explores this notion.

Valorization of Knowledge

While in the previous section my focus was on preservice teachers, here I will focus on children / school-age students and parents. For the last ten years I have been conducting research around issues of parents' views on the teaching and learning of mathematics (Bratton, Quintos, & Civil, 2004; Civil & Andrade, 2003; Civil & Bernier, 2006; Civil, Bratton, & Quintos, 2005; Civil, Planas, & Quintos, 2005). Throughout this research there is a recurrent theme that emerges in our conversations and interviews with families. This theme relates to immigrant parents' views on how their children are being taught in the U.S. versus the schooling traditions in their country of origin (which in my context is usually Mexico). As everybody else, parents bring their valorizations of knowledge to the discussion. Let me illustrate this point with an example related to different algorithms to show how this topic is of concern not only to teachers and preservice teachers.

All the parents we have talked to who learned how to divide in Mexico comment on their method being more "efficient" and "cleaner." A basic difference between the way they learned and the "traditional" approach to long division in the U.S. is that in Mexico they do not write down the subtraction, "we do it in our heads", and they only write down the result (the answer). This is what Marisol and Verónica said about the division algorithms:

> **Marisol**: When I looked at how he [her son] was dividing, he subtracted and subtracted and that he wrote all the equation complete I said, I even said, "this teacher wants to make things complicated. No, son, not that way! This way!" And he learned faster with this [Marisol's] procedure.
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> **Verónica**: I tried to do the same with my child with divisions, that he didn't write everything, but he says, "no, no, mom, the teacher is going to think that I did it on the computer." "You don't need to write the subtraction son," I say, "you only put what is left."… "No, no, my teacher is going to think that I did it on the computer, I have to do it like that." "Ok, you think that… but I want to teach you how we learned." And I did teach him, but he still uses his method, and that way he feels safe that he is doing his homework as they told him to. The same thing with writing above what they borrow and crossing it out, I tell him, "and I remember our homework could not have any cross-outs," whereas his does.

A topic of concern for many of the families we have interviewed is their perception that the level of education is lower in the U.S., often commenting that they thought their children were behind in mathematics compared to relatives or friends in Mexico.

> **Ernesto:** I think that the educational level, in the case of my son, the schools are very basic the level in Mexico is much higher. I'm saying that because I have nieces and nephews there and here and there, I see that they have learned more things at school…No, it's that he's [one of his nephews in Mexico] in fourth grade and my son is in fourth grade too. What they're giving my son now, he (the other child) learned in second grade. So, the educational level is lower and they learn more slowly than they learn in Mexico
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> **Bertha:** No, I'm not happy. I feel that there is repetition of a lot of things; I don't understand why the teaching is so slow, I don't like it, I don't like the system, I don't like it at all. I, when we go to México … my nieces and nephews or my husband's nieces and nephews, there are children that are more or less the same age as Jaime and I see that Jaime is behind. Here they tell me that Jaime (is) really excellent.

Researchers have made observations similar to those of our participants. McLaughlin (2002) suggests that Mexican students' mathematics background often exceeds the expectations they face when entering a school in the United States. We also have data from the children's experiences with the different educational systems. Lucinda, one of the mothers who was concerned with her daughter's schooling in the U.S. and wanted to also teach her the way she had learned in Mexico, commented that when they first arrived, her daughter was a third grader (8 years old) and was not very happy with the school in the U.S. because she said that it looked like play, "why, mijita?" asked her mother; "because they are making me do 4 +3, mom, I don't want to go this school. It's weird." And by "weird" she meant "easy."

Below is an excerpt from an interview with a sixth grader in 2001, when he had recently arrived from Mexico:

Researcher: Describe yourself as a math student

Student: I am advanced because in Mexico the schools are a year ahead. I am very fast at doing things. The teacher gives me harder work. (…)

Researcher: What is your best subject in math?

Student: Algebra

Researcher: You already know algebra?

Student: Yes

Researcher: Where did you learn algebra?

Student: The teacher [name of his current teacher] showed us. In Mexico, they had already taught me algebra. And the teacher here is barely starting to teach some algebra.

As part of our more recent work, we continue to study parents' and children's perceptions of the teaching and learning of mathematics, in particular among those who have experienced two educational systems (e.g., U.S. and Mexico), but we are also playing close attention to issues of language and how they affect students' learning of mathematics. The excerpt below is from an interview with a mother and her son (a sixth grader), about four months after they arrival to the U.S. This interview underscores the child's and mother's frustration at knowing the mathematics but not having the language (English) to participate or to fully understand the teaching:

Marta: So, I would like to know, if you can explain to me, if I went to your school in Mexico, when you lived in Mexico, what would I see in a math class? Tell me a little bit

Alberto: There, they teach things that here… there they teach you… they are ahead

Marta: They are ahead.

Alberto: And here, they teach me things too, things that they taught me there… but what they taught me there, I already know it here, it's just that here it's hard because of the English. (…)

Alberto's mother: What I feel is that yes, I notice that they teach them more things there. Now, here the difference is that you run into the language, because in this sense… That is, for them it's perfect what they are teaching them because in this way it's going to help them grasp it, to get to the level, because for them, with the lack in English that they have, and if to that we were to add, uh, what's the word? If they give them all the information, like a lot, very dense, too much teaching during this period, to tell you the truth, it would disorient them more. Right now, what he is learning, what I see is that it's things that he had already seen, but if he gets stuck, it's because of the language, but he doesn't get stuck because of lack of knowledge. (…)

Marta: So, you think that since he has already studied it in Mexico, the content, that this to a certain extent helps him

Alberto's mother: It makes it easier (…) Because he says, "ay, mom," he says, "and things that they ask and that they are really easy, and I get desperate because I want to answer, because I understood. And there are other things that I don't understand, but once I see the answer, I realize that I already knew it. … But I didn't understand the question. If I didn't understand the question, I cannot answer it, because I didn't understand them."

Alberto's mother thinks that it is a good idea that they are teaching him something that he already knows because he does not know English well yet and it would be too much for him to learn new content and English. Is this an equitable approach to the teaching of immigrant students? In Anhalt, Ondrus, & Horak (in press), the authors discuss an experience in which an instructor taught a mathematics lesson in Chinese to a group of middle school teachers. The teachers (most of whom were part of our Center CEMELA[2] and therefore, taught a large number of English Language Learners (ELLs)) realized the similarity in trying to learn in Chinese to their students' learning in English. Some of the teachers observed that because they were familiar with the mathematical content, they did not pay attention to the Chinese language and focused only on the mathematics. Teachers reported this was a powerful experience that made them think about the policy of student placement in their schools. It made them wonder about a common placement practice that places ELL students in lower level mathematics, the idea being that it will help them learn English. Teachers questioned whether through this practice students would learn neither English nor mathematics.

Are the educational needs of immigrants students being met by lowering the level of the content so that "they can learn the language"? This situation is not unique to the U.S. For the past several years, I have been collaborating with Núria Planas, a researcher in Barcelona (Spain) whose work focuses on the mathematics education of immigrant students in that city. Until 2000, immigrant students in public schools in Barcelona were placed in special classes with students with learning difficulties and physical disabilities. Currently, students with "language problems" (e.g., immigrants) are in a separate program for part of the day primarily for two subjects (mathematics and language). In that program they still work on the same adapted curriculum (as students with learning difficulties), which usually covers material two or three grades below their current grade.

In the first part of this section on valorization of knowledge I have focused mainly on parents' perceptions, and in particular immigrant parents, of the mathematics education their children are receiving in their "new" country. One could say that this is normal generational discourse—parents trying to show their children how they were taught because they feel that it was a "better" way. I argue, however, that these differences in approach take on a different light when those affected are low-income, immigrant families, whose knowledge has historically not been recognized or valued by institutions such as schools (Abreu, Cline, & Shamsi, 2002). This notion of their knowledge not being recognized or valued may even be more exacerbated if these students are given a lower level curriculum and made seem seen as "deficient" because they are not proficient in the language(s) of instruction. Planas (in press) looks into this situation by focusing on local students' perceptions of their immigrant peers' knowledge. In her research study, Planas interviewed twelve 15 and 16 year-old non-immigrant students from the same classroom in a high school in Barcelona that had a high percentage of immigrant students (60% of the students were from Morocco). In that particular classroom fourteen out of twenty-eight students were immigrants (Morocco, Dominican Republic, Pakistan, and Bangladesh). The school, as is the case with schools with high numbers of immigrants in Barcelona, is in a low-income neighborhood. Planas' research is particularly insightful in that it seeks to understand issues related to immigrant students in the mathematics classroom, not only from the point of view of these students, but also from the point of view of the "local" students (see Planas & Civil, 2002; Planas & Gorgorió, 2004). Planas' (in press) findings point to a deficit view on the part of the local students towards their immigrant peers. Attached to this deficit view is a lack of recognition and appreciation for the immigrant students' ways of doing mathematics. The "local" students point out that their peers' mathematics are different and these different forms of mathematics are not seen as useful or appropriate, as the quotes from two of these local students show:

> **Pau**: Their [immigrant students] comments help us make sense of the situations before starting solving the problems, but anyway, we cannot always start making sense of it like they do. Our maths are what they are. And theirs… they are fine, but sometimes they just don't fit in.
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> **Maria**: We are not in the classroom to learn their mathematics but to learn ours. That's what the exams are about. (…) I am not expected to learn Murshed's way of subtracting.

It is well known that there are different algorithms for arithmetic operations. My point here is not about "the" way to divide in Mexico vs. the U.S. or "the" way to subtract in Spain vs. Morocco. My point is about whose knowledge is being valued and how these different valorizations may affect students' participation in mathematics classes. This brings me to a key concept in my research—the notion of participation.

Does Everybody have a Voice?

> I try to understand and in class, I listen and ask questions but most of the time I have absolutely no idea what is going on. And what my peers say to me sounds like a dialect of the Alaskan Eskimo. [Carol, preservice elementary teacher]
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> There is hope yet when I can legally use my methods to solve a problem. [Vicky, preservice elementary teacher]

Carol and Vicky were students in the same section of a mathematics content course for preservice elementary teachers, for which I was the instructor. In this course I used a discussion-based approach, in which students largely worked in small groups on mathematical tasks that were often intended to create cognitive conflict (such as the proportional reasoning task alluded to earlier (Civil, 1989)). My approach (though not so clearly formulated at the time) was grounded on the idea of developing a community of learners in which students would feel comfortable questioning approaches and procedures they had taken for granted (e.g., why do we "invert and multiply" to divide fractions?). By encouraging different approaches to solving problems, I was aiming to open up the patterns of participation and, if possible, to undo the labeling that tends to classify learners as being "good at math" or "not good at math." My efforts failed with Carol, who often expressed her frustration at an approach that in her view exposed her as a failure to her peers and the instructor. She would have preferred a lecture-based approach, in which she was allowed to "remain anonymous." Vicky was also very anxious about her mathematics knowledge and kept on saying how she "couldn't do it using algebra" and would often look up to her peers who could use algebra. But Vicky became more comfortable participating in group discussions once she saw that her methods were accepted. Although she tended to label her methods as not being the "math way," the fact is that her approaches were often conceptually clearer than the more "traditional" ones her peers used. For example, for the following problem, "*If you need 1 1/3 cups of sugar and 4 cups of four to bake a cake, how many cups of sugar will you need if you want to use 7 cups of flour?"* Vicky drew the cups of sugar and flour and immediately saw the correspondence between 1/3 cup of sugar and 1 cup of flour, thus concluding that she would need 2 1/3 cups of sugar. Vicky seemed to approach the problem in what could be called a more informal way, using everyday type reasoning. The rest of her peers opted for an algebraic approach quite typical of how ratio problems are solved in school. Some of them became lost in the procedure, due to their difficulties handling mixed numbers.

This is just one illustration of the many examples that I have encountered in my work with preservice elementary teachers (and now more recently with parents), in which adult learners who often feel unsuccessful in school mathematics, bring in ways of reasoning that are clearer and more efficient than the school-based procedures. Certainly, the issue of how general are these informal / out-of-school procedures remains. Would Vicky have been able to solve the sugar-flour problem had the numbers involved been others? Or when is it appropriate for students to bring in their everyday knowledge? For example, Cooper and Dunne (2000) illustrate some of the problems that occur when students (particularly working class students) "import their everyday knowledge when it is 'inappropriate' to do so" (p. 43). But my interest in these out-of-school approaches is on their potential for the participation of more students in the learning process.

My interest in the concept of participation started with trying to understand the obstacles to participation in the sense of students not feeling confident in or not valuing their own approaches to mathematics because they were not the "school way." The large body of research on situated cognition and on out-of-school mathematics versus in-school mathematics is particularly relevant to my work (Brenner & Moschkovich, 2002; Brown, Collins, & Duguid, 1989; Lave, 1988; Nunes, Schliemann, & Carraher, 1993). Many of these studies document how successful and resourceful people are at inventing their own methods of solution to tackle tasks that they see as relevant in their everyday life. Yet, some of these studies also document a lower performance once a "similar" task is presented in a school context. To me, a key question is that asked by Hoyles (1991), "*is it possible to capture the power and motivation of informal non-school learning environments for use as a basis for school mathematics?*" (p. 149) (italics in original). This interest in bridging in-school and out-of-school mathematics and thus my interest in opening up the participation patterns moved from a somehow cognitive emphasis (as in an intellectual interest in different approaches to problems) to a more social and cultural emphasis when I started working in primarily low-income, Latino communities. I was struck by how resourceful and involved in the everyday working of the household some of the children were, while these same children were not particularly "successful" by school standards (Civil & Andrade, 2002). I was intrigued by what it would look like to try to develop learning experiences that would build on these students' (and their families) knowledge and experiences while ensuring that they advance in their learning of academic mathematics.

In Civil (in press) I discuss some of our efforts towards developing a mathematical apprenticeship in a school setting by embedding the mathematical learning in the "context of a sociocultural activity in which the pupils want to participate and in which they are able to participate given their actual abilities" (van Oers, 1996, p. 104). A construction module in a second grade class highlights my dilemmas at developing an approach to teaching and learning that emphasizes collaboration and engagement in activities that are important to the practices of the community (Lave, 1996; Rogoff, 1994), but also brings the mathematics to the foreground. The garden module in a fourth/ fifth grade classroom presents an example in which sociocultural practices are combined with cognitive approaches (e.g., enlarging a garden is followed up by a school task on exploring area of an irregular shape; task-based interviews are used to assess students' understanding of certain aspects of measurement).

Finally, there is another aspect of participation that was probably present all along in my work but has become more prominent in my recent research. It relates back to the concept of valorization of knowledge and whose knowledge is valued / recognized and when or where. We have looked at these issues in relation to the concept of norms (Yackel & Cobb, 1996), as in, for example, how immigrant students may be interpreting the norms differently from local students (Civil & Planas, 2004; Planas & Civil, 2002). In Civil (2002), I document a teaching innovation in a fifth grade class in which we tried to combine three forms of mathematics: school mathematics, mathematicians' mathematics, and everyday mathematics. Although some of the patterns of participation changed and opened up, overall the social and sociomathematical norms that were in place (and conflicted with those from our innovation) and the influence of status and students' perceptions of each other played a role in who had a voice when in the classroom. The "popular" students (which in that school often meant those in sports teams such as basketball or softball) and the students in the Gifted and Talented program had a voice in the mathematical discussions. As Lampert, Rittenhouse, and Crumbaugh (1996) write, "children do not readily separate the quality of ideas from the person expressing those ideas in judging the veracity of assertions" (p. 740).

Some Issues for Reflection

In this section I raise what I view are some issues derived from what I have presented so far in this paper. One such issue relates to the difficulties in developing mathematical learning experiences that while being true to the context (e.g., the construction or the garden modules) are also true to our mathematical agenda (Civil, in press). These difficulties, I argue, have to do largely with our values as to what we count as mathematics, as well as our own academic training that may make it harder to uncover the mathematics in everyday contexts. As a teacher in one of our study groups once asked, "if you have too much school mathematics, does it erase our practical mathematics?" In our work we have found the pedagogical transformation of community knowledge into school mathematics learning opportunities to be a non-trivial endeavor.

Students' views as to what they are willing to count as valid mathematics also play a key role in this process. Do students view the mathematics in the teaching innovations as "real mathematics"? Students may have indeed been involved in rich mathematical opportunities but if they do not see what they did as the mathematics they should be learning in school, or if the connections to what they may expect to see in the next grade are not made, are we helping these students? As I reflect on our efforts to try to bring change to the teaching of mathematics in these classrooms, I find Spradbery's (1976) work particularly relevant (even though it is 30 years old!). Spradbery describes the experiences of a group of sixteen-year-old students unsuccessful at school mathematics. Outside school, some of these students kept and raced pigeons. The author goes on to describe some of the mathematics embedded in this practice, and then writes:

> Although the mathematician may regard certain aspects of pigeon-keeping (along with many of the other daily activities of children) as being 'mathematical', such

> knowledge appears to have little value or status in the classroom. For 'Maths' to be 'Maths' (or 'proper Maths', as a number of children described it) it has to be separated from other everyday knowledge. (p. 237)

Spradbery (1976) describes the opposition among students—who had so far failed at school mathematics—towards an innovative curriculum that was intended to be more liberating by presenting situations for which the students were encouraged to use their own intuitions and knowledge. In our work, we also had students questioning whether what they were doing was mathematics. Currently, I often find myself wondering about innovations that try to contextualize the mathematics in situations that claim to be more relevant to the reality of the working-class Latino students with whom we work. Who decides what is relevant?

Another related issue in our work relates to the notion of norms (Yackel & Cobb, 1996). The teaching innovations we have tried to implement combine notions of communities of learners in which students engage with mathematics as mathematicians (Lampert, 1990) with notions of apprenticeship learning and other characteristics of out-of-school learning. The norms for these innovations had to be very different from the norms that were often in place (and well established) in the school where we conducted our work. Students resisted our approaches. One of the main obstacles we encountered with the fifth graders was their reluctance to engage in discussions about mathematical problems. As the teacher explained, "they [students] didn't see the point of the discussion; they didn't like waiting on everybody to talk. …They didn't feel like that was work. To them, work is filling out worksheets and turning the paper in and seeing if they got it right or wrong" (Civil, 2002; in press).

A Closer Look at an Example of Bridging In-school and Out-of-school Learning

I have briefly referred to some of the teaching innovations that we tried over the years and that are described in more detail elsewhere. Here I describe a teaching experience that took place quite a few years ago and that for many will seem like a trip to the past because it is based on the use of a Logo environment. What we used (i.e., Logo) is not the point here; what I want to do is to illustrate some of the issues raised earlier in this paper and provide an example of an experience in which we tried to develop a learning environment that captured some of the characteristics of out-of-school learning while pushing for the mathematics. The choice of Logo reflects my cognitive inclination, as it provides an environment in which with very basic few commands, students can right away explore mathematics. At the same time, our approach took into account the context, hence showing my socio-cultural inclination. This was a particularly difficult fifth grade classroom in which the academic agenda of the teacher (who was new to the school) clashed with the children's agenda. One of the researchers conducted interviews with each of the children to gain a better understanding of their social context. As she shared with me, "learning about what some of these children were going through in their daily lives made their resistance to change in the classroom quite understandable. Change for many of these children came as yet another form of upheaval in their already hectic lives." It was under these conditions that we tried to create a change to the teaching and learning of mathematics. A key feature of the learning innovation we envisioned was the sharing of ideas in a safe and supportive environment. Establishing this feature was a major obstacle in this classroom, given that the class was dominated by a core power group of five boys, who through popularity and intimidation manipulated the classroom dynamics.

We introduced the use of Logo to the whole class and after a few sessions, we offered the option to continue working in the computer environment. A group of eight children expressed interest in doing this (but one dropped after two sessions). The rest of the class (17) stayed in the regular classroom with the teacher. With this group of seven children (who were representative of the diversity in the classroom) we set out to develop an environment that would have some of the characteristics of out-of-school learning, which we had identified from the literature (Brown, Collins, & Duguid, 1989; Hoyles, 1991; Lave, 1988, 1996; Rogoff, 1994), namely: 1) Learning by apprenticeship; 2) Working on contextualized problems; 3) Control remains largely in the hands of the person working on the task; and 4) Mathematics is often hidden; it is not the center of attention and may actually be abandoned in the solution process. Rogoff (1994) discusses three models of teaching and learning. In the transmission model, knowledge from others is passed on to the learner (adult-centered) and in the acquisition model the learner discovers the knowledge on her or his own (child-centered). In the participation model, the learner participates in a community of learners. In this model, learning takes places through collaboration and engagement in activities that are important to the practices of the community. I argue that this group of seven students learned through participation in a community of practice that emerged in the computer lab. Logo was new to all students in this class. In a sense Logo had an "equalizer effect" that may have allowed for a fresh start for all the students who stayed in the group. They were able to put aside their roles and labels. It allowed for children who had been labeled as being "less successful" in academic subjects to shine as they demonstrated their expertise and as they followed their own inquiries. Students who barely spoke to each other in the regular classroom started trading discoveries about Logo. For example, one student became an expert at using color; another became an expert at writing procedures on the flip side (this is the area where procedures are written, as opposed to immediate mode programming); another student, using his prior knowledge of the computer system, explored all the different "gadgets" included in Logo. What we soon noticed was that students were constantly sharing ideas and were well aware of who knew what. We saw "learning traveling through the lab" and students learning through interactions with their peers. It is in this sense that we believe that what took place in this Logo group had some of the characteristics of learning through apprenticeship. This idea of apprenticing is also addressed by Moll and Greenberg (1990), who write, "it is when the content of interactions is important or needed that people are motivated to establish the social contexts for the transfer or application of knowledge and other resources" (p. 326). I next give a glimpse of the dynamics in this group with a focus on the mathematical explorations.

A Mathematical Community of Practice?

While working on a task on how to draw a hexagon, two students (Daniel and Ben) found a way to make a variety of star polygons. Soon thereafter, two other students (Jennifer and Jorge) became interested in drawing star polygons. We discussed with Jennifer and Jorge some of the mathematics involved in this task. From a previous discussion on turns (grounded in part on several students' knowledge about skateboarding), these two students were familiar with the notion of 720 for two whole turns. They then found 720 ÷ 5 and tested the procedure: **repeat 5 [fd 40 rt 144]**, which produced a five-pointed star. Following this, Jorge started working on looking for divisors of 720 using, at times, the calculator available in the computer. For example, he came up with: 720 = 72 x 10. He then typed: **repeat 10 [fd 40 rt 72]**, except that this command did not produce a star polygon. On his own, Jorge continued exploring and found: **repeat 9 [fd 70 rt 80]**, which does produce a star polygon. Through a process of mathematical calculations and manipulation of the commands, Jorge found combinations that produced star polygons and recorded them on the flip side.

As Jorge and Jennifer were working on star polygons, the question of "how to draw a circle" arose. Several of the students became interested in this challenge. On the blackboard, Jennifer drew a 90º angle, and then a 40º (exterior) angle. She then said, pointing to her drawing, "it's going to have to be a very subtle turn, a subtle angle." She had the correct image of turning minutely (a subtle turn) every time, however, she did not quite know how to do it on the computer. Ben, meanwhile, did get the turtle to draw a circle. In response to Ben's accomplishment, which came as a result of step-by-step commands, our challenge to him was to draw a circle in one single command. Though the challenge was made to Ben, it was Jorge who took it up and explored with the template **repeat __ [fd 1 rt 1]**. After trying several inputs, 360 clicked all of a sudden. Sara, meanwhile, who was not sitting near this group of students, had also figured out how to make a circle and quickly shared it with Carolina and Judith who ended up incorporating a procedure to draw a circle as part of their final projects. Daniel also ended up working with circles for his final project because he became interested in creating a spiraling tunnel. To do this he needed circles of differing sizes, which led to his learning about variables in Logo. Daniel ended up with this procedure:

```
To c :n
 repeat 180  [fd :n rt 2]
end
```

In order to get the desired effect of increasingly larger circles within close proximity to each other, the need for decimals arose. We were not aware of what these children knew about decimals. We suggested that he try c .7, c .8. Daniel explored with these numbers. The first "surprise" came up when, after typing c .9, he typed c .10 and a smaller circle appeared. He then tried c 10, which gave him a much larger "circle" (actually, it did not look at all like a circle!). Following this, we drew a number line on the blackboard and asked Daniel to place .9 and 10 on a number line. He appeared to be confused by this question. Jennifer, who was working on something else but sitting near the blackboard, became interested in the conversation and helped him out. Daniel seemed unclear about what decimals were. He was not sure about what "that point," as he called it, was doing. We then switched to decimal fractions, since the students had worked with these in class. This seemed to help, but it was difficult to tell whether there was understanding or rather pattern recognition taking place. In looking at 12/10, 14/10, Daniel quickly supplied 13/10 and 11/10. His final project, however, went back to the decimal notation, this time in an increasing sequence from .1 to 1.9 (by increments of .1). In the presentation of their Logo projects to the whole class, students were intrigued by Daniel's procedure "**c**" and the various inputs. They asked Daniel to try c .50 and were surprised by the fact that it was the same as c .5. They then suggested c .47, expecting something bigger, and again were puzzled with the outcome. Looking back at Daniel's work, we think that similar scenarios could be turned into an opportunity to explore students' understanding of decimal numbers.

I would like to emphasize two salient features of the work with these seven children, at the social and cognitive levels. The social behavior governing our work in the lab with these seven children was a complete change from that present in the regular classroom. While in the lab, we did not witness any put downs or negative comments directed at each other or at us (several of these same children did not get along in the regular classroom). Students behaved in a very polite manner, informing us when they were leaving the room, and being overall very cooperative. This occurred in a natural way, since we never talked about what behavior we expected from them. We focused on their work on the computers. The students seemed relaxed, happy to be in the lab, and while quite individualistic in their work on the computers, there was considerable chitchat and sharing of ideas.

Engaging with the students in conversations about their work was difficult. The teacher had mentioned to us that many of the children in the classroom did not seem to trust adults and were not willing to converse. An added difficulty, perhaps, was that we were trying to dialogue with them on an academic topic. As we tried to have them tell us more about what they wanted to do or what they were thinking, we were often met with silence or comments we could not quite follow. But with most of them, we succeeded in engaging in a conversation on their problem solving strategies for their project. For example, with Jennifer, it was on patterns; with Daniel, decimals; with Jorge, how to make sure the football field fit on the screen and was an accurate (to scale) representation; and with Carolina, how and where to put the moon and how to obtain a visually aesthetic effect "more efficiently." The cognitive gains became clearer as the students presented their projects to the class as a whole. The Logo students were knowledgeable and comfortable with the language of the Logo environment. Each child spoke as an expert, demonstrating considerable command of the situation and clarity in the presentation, in what was for many of them a less than receptive audience (i.e., the reality of the classroom).

A Look at Our Current Work

The Logo project I just presented allowed me to experience some of the theoretical constructs such as characteristics of out-of-school learning, communities of practice, mediation, and constructivism in the context of students engaging in mathematical explorations. That work underscored the importance of understanding and paying attention to the social context. In our current work we are determined about the concept of context as we take a holistic approach to the mathematics education of working-class Latino children: parents, teachers, children are central to our work. Most of my research at the moment takes place at a school that is 90% Latino, 26% English Language Learners, and with 95% of the students eligible for free or reduced lunch (the average for the state of Arizona is 49%). At this school we have several research activities in place, including: 1) a teacher study group aimed at teachers reflecting on their practice; teachers explore mathematical content for themselves, as learners, but also reflect on students' work, and engage in discussion around language and mathematics; 2) classroom visits to not only observe the mathematics instruction but also to support the teachers and students; 3) a parental component in which parents take part in mathematics workshops (sometimes with their children and facilitated, in part, by their children's teachers); 4) an after-school mathematics club in which children are encouraged to work on contextualized mathematical projects (e.g. a garden; getting to know your community) and where both languages (English and Spanish) are used (bilingual education is severely limited in this state, thus children have few opportunities to engage in academic discourse about mathematics in Spanish during their regular school hours). All these research activities emphasize our focus on teachers, students, and parents.

A focus on teachers

The mathematics curriculum in place at this school is "reform-based" and the teachers with whom we work are working hard at implementing it. The curriculum is relatively new for most of the teachers and, as with any curriculum, it portrays a certain view of what it means to do mathematics. In some aspects, that view is not very different from what teachers at this school have been engaged in throughout their participation in professional development experiences.

For example, in these experiences they learned about group work, hands-on materials, open-ended problems, and teaching for understanding. But, as I discuss in Civil (2006), some of these professional development efforts aimed at helping teachers teach using a "reform-approach" leave me wondering whether teaching mathematics was becoming a smorgasbord of activities with no apparent road map. At this school the fact that teachers are expected to use a specific curriculum provides to a certain extent this road map that I saw lacking at other places, years back. This is not to say that it is unproblematic. I am particularly intrigued by how teachers make sense out of the curriculum and how they decide on which tasks to implement and what that implementation looks like. The work of Stein, Smith, Henningsen, and Silver (2000) on the cognitive demands of tasks is particularly helpful to my analysis of classroom teaching at this school. But in addition to the content demands of the tasks, I am also interested in what kind of support (affective and cognitive) the teacher and students create in the classroom to encourage the mathematical participation of all students. For this area, Empson's (2003) use of participant frameworks in her analysis of two low-performing students is quite relevant as I look at interactions in the classroom. And a third area of interest is what view of what it means to do mathematics is being conveyed or co-constructed in these classrooms.

I am currently in the middle of analyzing data from several videotaped lessons from a fourth/fifth grade classroom with my focus being on mathematical discourse and in particular on students' reasoning. These three areas of interest I just mentioned (nature and demands of tasks; support; view of mathematics) are closely intertwined in my analysis. For example, one of the lessons focused on solving word-problems on multiplication and division. The students worked in groups and for each problem they had to 1) demonstrate how they solve the word problem; 2) write an equation; 3) solve; 4) explain the process. One of the problems was "*a restaurant serves different types of sandwiches; it has four different types of meat (turkey, ham, baloney, and roast beef) and three different types of cheese (Swiss, Cheddar, and Jalapeño). How many different sandwich combinations can the restaurant sell?*" Students approached this problem in a variety of ways, using several different representations in their solutions. All the students but two interpreted the problem as "expected," thus leading to 12 different kinds of sandwich. These two students, who were working together, tried to find different combinations, that is, with 2 kinds of meat and 1 cheese, 3 kinds of meat and 1 cheese, or 2 kinds of meat and 2 types of cheese, and so on, making it a much more demanding problem. The teacher encouraged the different groups in their work, as she walked around the room. She then asked some of the groups to present, and as they presented, she asked them questions that related to the idea of making sense "why did you choose to multiply?", or "why did you decide on that equation?" When a group presented their work on a different problem in which they had first divided incorrectly (and they showed that incorrect way), the teacher said "you see, they did it four different ways, and three didn't make sense to them, but they kept with it and they got it." And later on, she said, talking in general about the different ways to solve a problem, "remember, you never give up; look at all the different ways you could do it." It is true, however, that the teacher seemed somewhat at a loss with the two children who approached the sandwich problem by looking at combinations differently form all the other students. But although she did not really probe these two children much on the mathematics, she encouraged them to pursue their thinking and invited them to present it to the class. She certainly offered them (as well as the other students) affective support and to an extent cognitive support. Through her emphasis on making sense, looking for and at different ways, persistence, this teacher is trying to convey a view of mathematics as an area of inquiry and creativity.

Our analysis of classrooms is only a piece of this holistic approach in which we try to understand the interactions of the linguistic, cultural, social and political contexts with the teaching and learning of mathematics by paying attention to the children, their parents and their teachers. In the next two sections we focus on parents by looking at parent-child interactions around arithmetic, and we focus on students, by presenting an incipient case study of a child to illustrate how we are looking at children as learners of mathematics.

A focus on parents

In Civil, Planas and Quintos (2005), we use a Bourdieuian perspective to interpret parents' perspectives on their children's mathematics education. In that article we argue for the need to know more about students' social contexts and in particular about their parents' perceptions of their children's mathematics education as part of our efforts to gain a better understanding of students' performance in mathematics. As Marisol, a mother in a previous research project tells us, "parents and children come together." Thus, at this school we are working on this idea of parents and children coming together and, based on a prior research project in which parents and teachers worked together, we are also building on the lessons learned from that experience (Civil & Bernier, 2006) and bringing in the teachers.

A particular emphasis of this school's curriculum is the development of flexibility when working with numbers. For example, in the $4^{th}/5^{th}$ grade class, to multiply 23 by 14, a student may do (20 + 3) (10 + 4), while another student may do (5 + 5 + 4) (10 + 10 + 3)). Some of the students seem to enjoy coming up with quite complicated and, I would argue, rather inefficient ways to break the numbers. But they appear to enjoy doing this (and in some cases, when I have asked them, they are aware that there are more efficient ways to break apart the numbers). Do they do it for fun? Or do they think that that is the goal of the activity, to come up with "complicated" ways to break apart the numbers? If that is the case, what view of mathematics are they developing? How are the tasks being interpreted is a question that I find myself asking, and not only about the students but the teachers too, as some of these approaches to doing mathematics are new to them also. How tasks are being interpreted by the different parties involved is particularly important in classrooms that are trying to implement a different approach to mathematics teaching and learning, as Lerman and Zevenbergen (2004) point out:

> Bernstein (1996) is detailed in explaining how power and control are translated into different pedagogies; the implications are that if students are to be successful they need to recognise the unspoken, or invisible, aspects of some pedagogies, particularly reform ones, as we discuss later. Two important considerations need to be made; one is how tasks are framed for students—the issue of contextualisation and recontextualisation--, and the second is how they are answered by the students—the issue of recognition and realisation rules (p. 29).

In our workshops with parents and children one of the goals is to introduce the parents to these other ways to do arithmetic. So, for example, at the $2^{nd}/3^{rd}$ grade level, to do 23 + 46 +7, children may do 20 + 40, then 7 +3, and finally add the 6 to the prior result. What we are currently analyzing shows the parents trying to teach their children the way they were taught (the "traditional" algorithm) in a very procedural way, with an emphasis on how to write it "correctly." For example, in the addition 23 + 46 + 7, a father showed his daughter how to write it vertically and put a 0 in front of the 7 to keep the numbers lined up. In another case, I was working with a mother and her son; I explained to her (based on the handout the teacher had given them) a way to do 51 – 22 by first doing 30 – 22, 8, then adding the 21 we were missing from the 51. I next suggested to her that she try this strategy on 42 – 13; instead, the mother went back to 51 – 22, set it up vertically and started explaining in to her son "if you have 2, to get to 10"; she wrote "8" but then realized that that is not what she wanted. I brought up again the strategy I just showed them, and she said, "but to me, it's easier this way" [pointing to the vertical set up she had written on the paper]. She then walked her son through the "standard" subtraction procedure, "when the number on top is smaller, you ask for 1 from the one next to it, …". The mother walked him through 51-22 and then encouraged him to try 42 –13. The mother helped him make the "2" into "12" and then the child asked, "do I put a 5?" pointing to the 4 in 42. The mother said very calmly, "no, no 3." After this, I tried to explain to the mother one of the approaches to subtraction that they were using in her son's classroom, showing her how they start at 13, then they may jump to 15 (by adding 2), then to 20 (by adding 5), then maybe to 40 (by adding 20) and finally to 42 (by adding 2), and then they add all the jumps to find the answer. As I was explaining this, I remember thinking "she is probably wondering, why are we teaching this, when her method is so much quicker." We are in the preliminary analysis of these interactions but we can already see how different views about teaching and learning mathematics are in play and can potentially come into conflict, even with the parents who are coming to the workshops and therefore being exposed to how and why their children are being taught this "different" way.

A focus on learners: The case of Julián

Julián was born in the U.S. but his parents are from México and he speaks Spanish at home. By fifth grade he had attended five different schools, 2 in México and 3 in the U.S. Part of the reason for the change in schools had to do with his family moving, but another part was his not feeling comfortable and thus changing schools, "the first school I went to was, Kindergarten, and there were these kids that always called me names. It hurt me. And my teacher never understood me" [interview, February 2006]. When referring to a more recent school, from which he also moved out to come to "our" school), he said, "she [the teacher at that other school] embarrassed me. … Sometimes she was okay, but, but then when I asked questions, she said, 'why are you asking the same question over and over?' And then, that's when, ah, when she embarrassed me, and I didn't understand it. 'Well, you should, you should if you were paying attention,' and I was."

When asked about his perception as a student of mathematics in the classroom, he placed himself as third best. He takes his work very seriously: his homework is carefully written; during the scale-drawing project in the after-school mathematics club, he inquired about whether they were going to also make a three-dimensional model, which would then involve measuring the height of the walls. The facilitator said that it was up to them; Julián then told one of his peers (who did not seem interested at all in doing it), "you don't need it, I **have to**, I want to do the model." In many ways, Julián is a "school boy"; he is quite good at following the rules of school and following what the teacher tells them to do. On the problem-solving day I referred to earlier (the sandwich problem), another problem they worked on was: "*this year Mark saved $420; last year he only saved $60. How many times as much money did he save this year than last year?*" Julián and two other boys, Alberto and Leo, worked on this problem together. Alberto, who often seemed to be off task or claimed being "lost," right away said, "so, this is a division problem." Julián first said yes, then no and then said, "we need to subtract." As their first step on their paper they wrote the subtraction (420-60=360). Alberto, “we’re done”; Julián, “no, we are not… for an equation.” They seemed unsure as to what to write for an equation (this is a topic that I am currently trying to understand better, as the writing of equations seems to be very important for this teacher; it brings back the issue of how, in this case the teacher, is interpreting the task (i.e., why did she want them to write an equation for each of these problems?)). They finally settled on \$420 - \$60 = N for their equation and Julián moved to the third step of instructions on the board (1) demonstrate; 2) write an equation; 3) solve; 4) explain your process) and says:

Julián: solve it [as if reading the instructions]
Alberto: we already solved it.
Julián: we did, but how can we demonstrate it, yeah, we solved it up here (points at the subtraction) but…
Alberto: just say that (points at the subtraction)
Julián: that’s all we did

While Alberto clearly indicates that they are done with the problem, Julián looks worried as he feels that what they have done does not reflect the four steps they were asked to have; they have numbered their steps and they only have two. This episode brings up many issues: what was Leo’s role in all of this? He does not say anything, though appears to be following what his classmates are doing; Alberto had the right idea, to divide, yet Julián’s is the one that prevails, why? Alberto seems willing to let go of the instructions the teacher has given them, as he considers they have solved the problem, while Julián struggles to make sense out of the directions, in particular “demonstrate” versus “solve.” Does he want to have the four steps because that is what the teacher is asking for? Does he realize that depending on how they approach the problem, some of these steps may be unnecessary? How does Julián decide when to follow the rules and when to “challenge” them?

Julián does seem aware of the school game and the artificiality of school tasks. To support their exploration of factors, prime numbers and other elementary number theory concepts, I presented a problem that involves machines that stretch bubble gum, where machine n stretches a piece of gum to a length n times its original length. So, for example, if I wanted to stretch a one-inch bubble-gum stick to 30 inches, and the machine 30 is broken, I could use machine 6 followed by machine 5. Julián right away said to the whole class, “but it’s still the same amount of bubble gum.” He is right; the way the problem is worded, these machines stretch the lengths of the sticks of gum but do not increase the amount of actual gum. Julián understood the problem and was able to offer different combinations for the various numbers I gave them. He knew how to interpreted the task from a school point of view. I doubt that Julián in a formal assessment situation would let his everyday or common sense interpretation “interfere” with his performance (Cooper & Dunne, 2000).

To a certain extent, in the after-school mathematics club we try to develop an environment in which school and everyday mathematics are brought together along the lines of my previous research. Because we see these children as mathematics learners, both in the school setting and in the after-school, we can address some of the issues brought up by Frankenstein and Powell (1994) in relation to the separation (and maybe even opposition) between everyday and academic knowledge. We can see how a concept such as that of scale, which is often studied in school mathematics, is applied to a task that while school-based (e.g., making a scale drawing of their classroom), because it is done in a somewhat informal setting, takes on a different flavor. As mentioned earlier, Julián took the scale-drawing project very seriously. At one point in that project the facilitator (a university researcher) is working with another student using a drawing that he (the facilitator) had made of the room showing some dimensions. Julián is working on his own sketch. All of a sudden Julián looks over their work and asks about one measurement that they have on the facilitator's sheet. From there, a conversation follows in which Julián challenges the facilitator's drawing and tells him he has the wrong measurements for one of the sides, "yes [takes his pencil and starts drawing on the facilitator's sketch], from here to here, it has to go till here, you didn't draw it correctly; they have to be the same [he then starts pointing to the walls in the classroom], look they are the same." And after that, Julián just goes back to his sketch.

One of our goals in this research is to focus on Latino, low-income students as powerful thinkers and doers of mathematics, in opposition to the deficit approach that is often used to describe these students. In the scale-drawing project, we see Julián as a confident student, immersed in the task, while offering suggestions to one of his peers and engaging in a content-based conversation with the adult facilitator. Crucial to capturing the case of Julián is the affective aspect (uncovered through rapport and interviews), Julián in the classroom (and hence the teacher's role), and in the after-school setting, as a place to pursue our understanding of Julián as a learner of mathematics.

Moschkovich (1999) points out that we do not know enough about the participation structures in the home cultures of Latino students in our local contexts. Furthermore, Moschkovich (in press), in her analysis of the potential contributions of non-mathematics education studies to the study of bilingual mathematics learners, writes, "sociolinguistics also suggests that analyses of classroom communication should be informed by data on students' experiences, building profiles of students' language history, educational background, and attitudes towards bilingual communication for students, peers, teachers, and parents" (p. 29). Through our current research we hope to address the issues that Moschkovich raises. We are developing case studies that cut across different activities and people and use multiple sources of data: video-tapes and field notes of classroom and after-school mathematics club sessions; interviews (affective / perceptions and cognitive) with children, teachers, and parents; observations and video-tapes of parents' workshops. Our goal is to try to capture the experience as learners of mathematics of a few of these children by looking at them doing mathematics in 1) the regular classroom; 2) the after-school mathematics club; 3) (if possible), out-of-school activities; 4) Through their parents' eyes or with their parents. This research is allowing me to bring together my cognitive and my socio-cultural interests to hopefully gain a better understanding of the complexity of what it means to teach and learn mathematics.

Endnotes

1. Parts of this paper are adapted from Civil, 2006.

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